Chapter 13

ELECTRIC CIRCUITS, BOTH AC AND DC

In the last chapter, we began talking about what it takes to make charge flow through a circuit. We are about to delve into the mechanics of that operation more completely.

A.) The DC Power Supply:

1.) There are two general kinds of charge flow possible in an electric circuit. They are *direct current*, generally referred to as *DC*, and *alternating current*, generally referred to as *AC*.

2.) Whether you are looking at AC or DC, a power supply's task is to provide energy to a circuit.

a.) It does this by creating a *voltage difference* from which an *electric field* is generated.

b.) In the presence of the electric field, charges move.

3.) The voltage provided by a DC power supply can be fixed at a particular value or can be varied, but it always motivates charge to move in one direction and one direction only.

a.) Sometimes called a *DC source*, the power supply's various circuit symbols are shown in Figure 13.1a.

b.) The "size" of a DC power supply is normally characterized by a single number (e.g., a *6 volt* battery). This denotes the voltage *difference* between its two terminals.

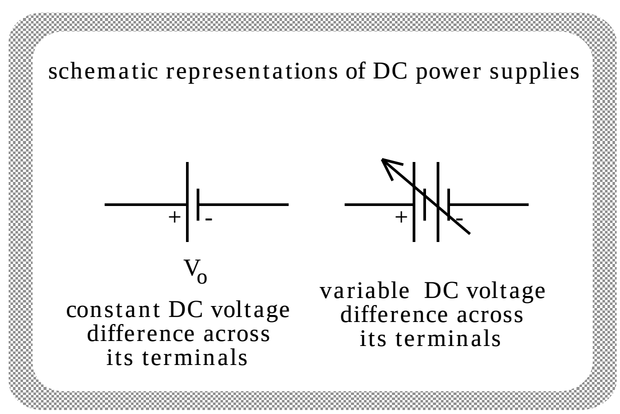


FIGURE 13.1a

c.) The low voltage terminal is usually black.

i.) Because it is the voltage *difference* that matters, this terminal is usually associated with *zero* volts.

ii.) The low voltage terminal is often referred to as the *ground terminal* and always is associated with the symbol of a negative sign (i.e., "-"). See Figure 13.1b.

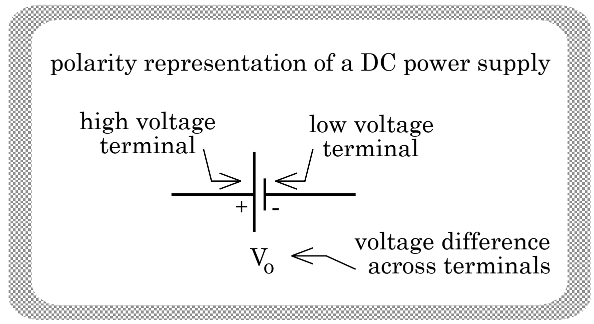


FIGURE 13.1b

d.) If the low voltage terminal is associated with zero volts, the high voltage terminal is associated with the voltage value of the source.

i.) The high voltage terminal is often referred to as the *hot terminal* and is always associated with the symbol of a positive sign (i.e., "+").

4.) Meters that measure DC voltage are called *DC voltmeters*.

a.) If you want to determine the *voltage difference* across any DC circuit element, hook the leads from a DC voltmeter to either side of the element and the meter will make the reading (see Figure 13.2).

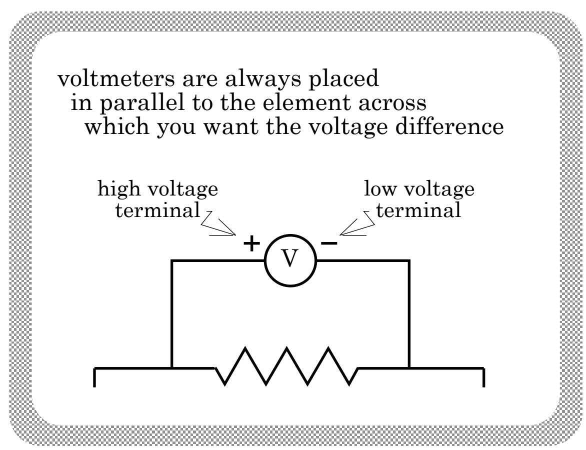


FIGURE 13.2

b.) A DC voltmeter is a polar device in the sense that it has a *high voltage* and a *low voltage* terminal. If you inadvertently hook the *high voltage* lead to the *low voltage* terminal, the voltmeter's needle will swing in the wrong direction.

c.) Although this will make more sense later, voltmeters have an enormous resistance to charge flowing through them.

B.) The AC Power Supply:

1.) The voltage provided by an AC power supply always varies in both magnitude AND direction. That means the electric field it sets up changes continuously, motivating charge to jiggle back and forth in the wires of the electric circuit to which it is hooked.

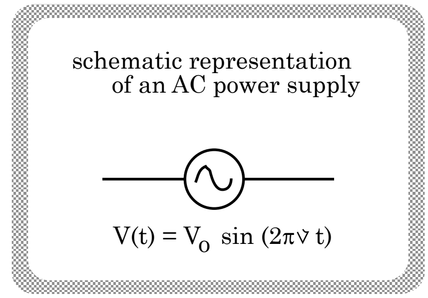


FIGURE 13.3

a.) Sometimes called an AC source, its circuit symbol is shown in Figure 13.3 (the sine function identified in the sketch will be explained shortly).

b.) Although it is possible to generate AC voltages that vary in odd ways (square waves, ramp waves, saw-tooth waves, etc.), the *voltage difference* across the terminals of any AC sources *we* use will vary in time as a sine wave.

i.) This means a *positive* voltage difference will correspond to charge flow in one direction while a *negative* voltage difference will correspond to charge flow in the opposite direction (see Figure 13.4).

c.) The frequency of the charge jiggle in a wire is dependent upon the frequency of the AC source.

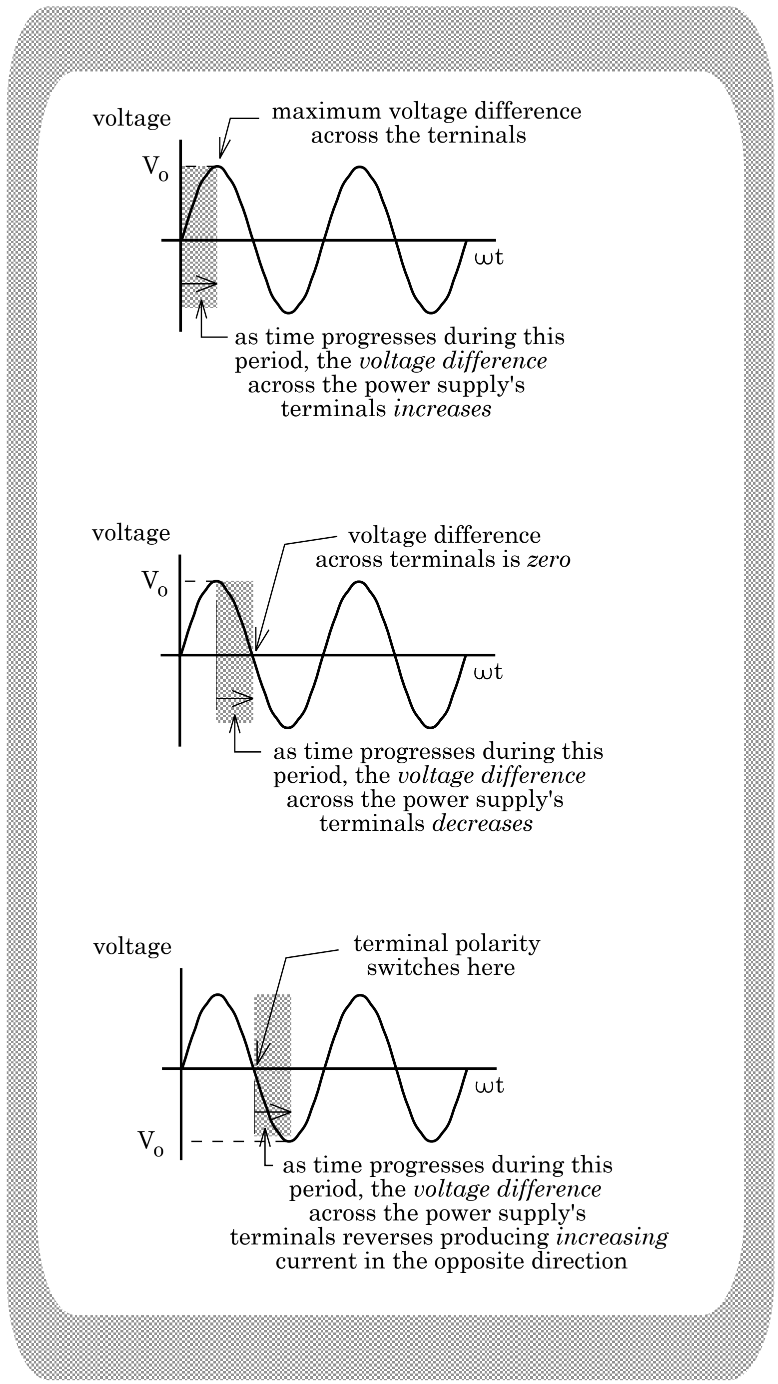


FIGURE 13.4

i.) An everyday wall socket provides power at 60 hertz. That means there are 60 times per second during which the voltage difference is a maximum with the field directed in one direction, and 60 times per second during which the voltage difference is a maximum with the field directed *in the opposite direction*.

ii.) Put a little differently, an incandescent light bulb that is plugged into a wall socket will turn on, then off 120 times per second (see Figure 13.4a).

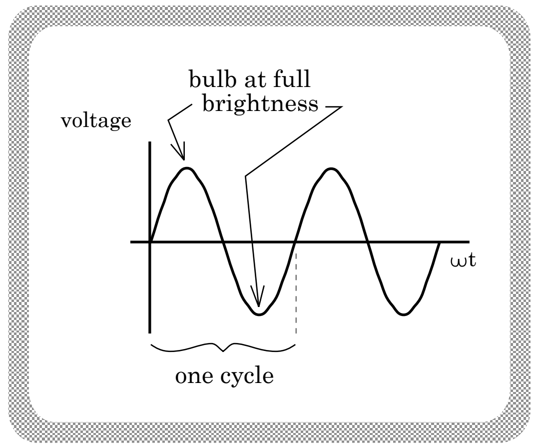


FIGURE 13.4a

iii.) We will occasionally use power supplies whose frequencies can be varied from 20 Hz to 20,000 Hz (this is the range of sound wave frequencies your ears can sense, assuming you haven't ruined them by now). These are called *audio sine wave generators* or *function generators*.

d.) Though there are AC voltmeters, what they measure is a little obscure and will be dealt with later.

C.) DC Current:

1.) Current in a DC circuit is defined as the amount of charge that passes by a particular point in the circuit *per unit time*. THINK ABOUT THIS.

a.) The units for current are *coulombs per second*, or amperes (normally shortened to *amps*).

b.) The algebraic designation for current is i.

c.) It is important to realize that current is a measure of the amount of STUFF (as in, charge carriers) that passes by per unit time. You don't *use up* current.

i.) If there are *5 amps* of current passing through a particular section in a circuit, assuming there are no junctions within the section, *5 coulombs of charge* MUST PASS BY EVERY POINT in the section *per second* (see Figure 13.5).

2.) A *branch* is defined as a circuit-section in which the current is the same.

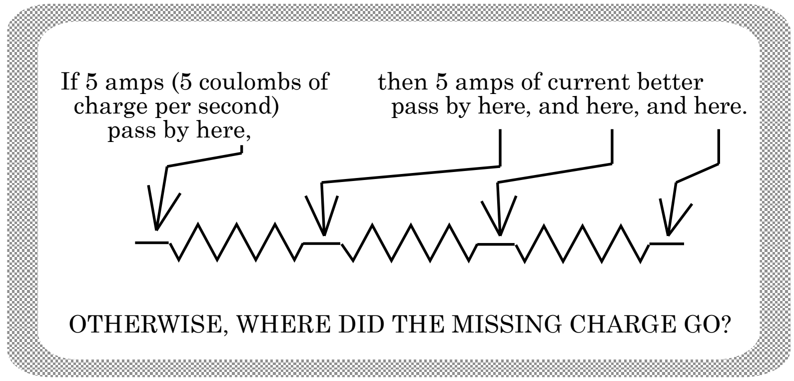


FIGURE 13.5

a.) Ignoring the fact that you have no idea what the circuit elements shown in the circuit stand for, count how many branches there are in the circuit shown in Figure 13.6.

b.) If you counted correctly, you found *six*.

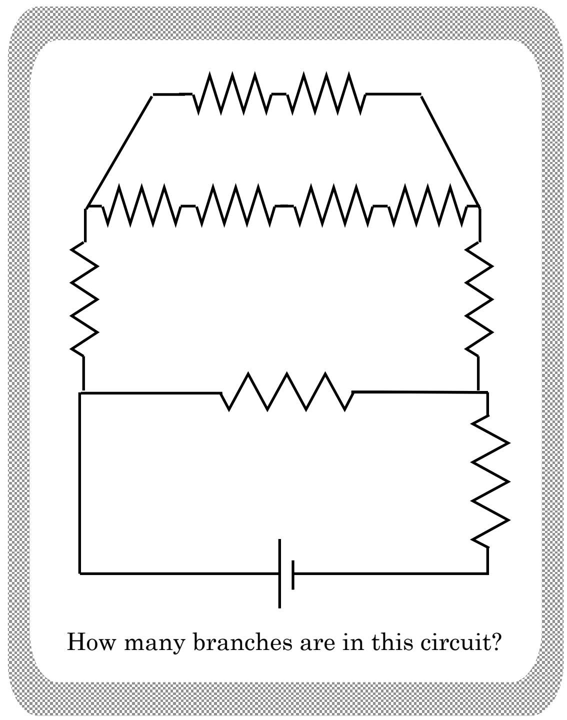


FIGURE 13.6

3.) Meters that measure DC current are called *ammeters*.

a.) If you want to determine the current through a branch, place a DC ammeter directly into the branch (see Figure 13.7). In that way, current will pass through the meter.

b.) A DC ammeter is a polar device in the sense that it has a *high voltage* and *low voltage* terminal. If you inadvertently hook the *high voltage* lead to the *low voltage* terminal, the ammeter's needle will swing in the wrong direction.

c.) There are AC ammeters, but what they measure is a little obscure and will be dealt with later.

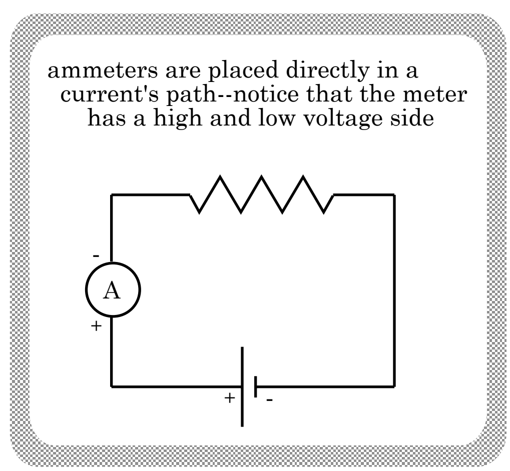


FIGURE 13.7

4.) You would think that the *direction* of a DC current would be obvious. After all, electrons move *opposite* the direction of the electric field that accelerates them, so DC current should be defined as moving from *lower* to *higher voltage* (i.e., remember, electric fields point from *higher* to *lower* voltage).

In fact, this *is* the direction of what is called *electron current*, but it is *not* the direction that is typically associated with what is classically called *current* (see Figure 13.8).

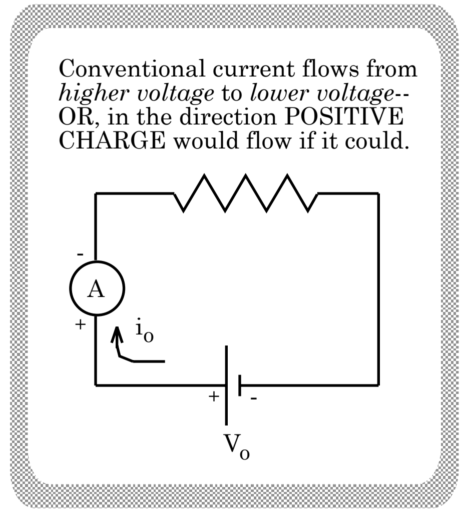


FIGURE 13.8

a.) This point of contention was inadvertently created when Ben Franklin (better know as *Ben the dude* by his surfing buddies), not knowing if it was positive or negative charge that moves in a circuit, assumed it was *positive charge* that flows.

b.) Franklin defined "current," as he called it, to have the direction that *positive charge* would flow, assuming positive charge *could* flow in a circuit. (This is now more formally referred to as *conventional current*, but in the United States the simple term *current* is also commonly used.)

c.) Is this obscure? A bit. Why haven't we changed it to fit reality? For two reasons. First, the people who have the power *to* change it (i.e., physicists) learned it that way and see no reason to make a change. And second, it sort of makes sense to assume it is positive charge that moves in circuits, at least from a theoretical standpoint. How so?

i.) How is the direction of an electric field defined? It is defined as the direction a *positive charge* would accelerate if put in the field.

ii.) What "falls" downstream along electric field lines as it moves from *higher* to *lower* voltage? It's *positive charge* that does that.

iii.) All of the theory is set up predicated on the idea that it is *positive charge* that is important. Why should you be surprised that Franklin would continue the tradition and assume it was *positive charge* that moves in an electrical circuit?

iv.) To put even more nails in the coffin, many *circuit symbols* reflect this assumption. A diode is a circuit element that allows AC current to flow in only one direction (remember, charge carriers in an AC circuit usually move one way, then the other way). The symbol for a diode is an arrow and a blocking line (i.e., a vertical line drawn to suggest a *stop*).

In what direction does the arrow point? It points in the direction *positive charge* would flow if it could flow in the circuit. It points in the direction of *conventional current* (see Figure 13.9).

v.) And when we want the direction of a magnetic field down the axis of a current carrying coil, we assume it is positive charge that flows and use a *right hand rule* to determine the field's direction.

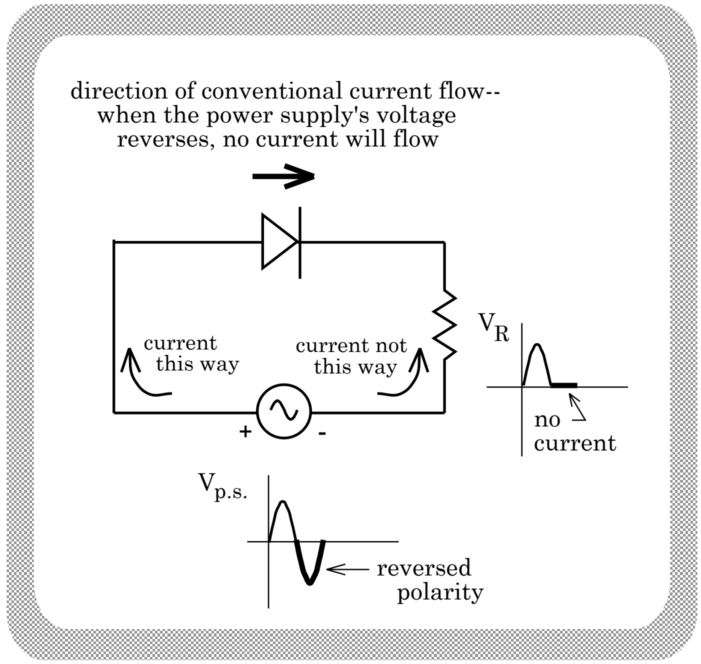


FIGURE 13.9

d.) If this was a normal physics class, I would have defined the *direction* of current flow in a DC circuit to be the direction *positive charge* flows in a DC circuit (assuming positive charge *could* flow in the circuit). In doing so, I would be defining *current* in consonance with almost all schools in the United States and Britain. You, the student, would grumble for about twenty minutes over the idiocy of the definition, then accept it as hundreds of millions of physics students have done over the course of the last few centuries, and get on with it.

e.) The only reason I am not doing this but, rather, making a big deal out of the difference, is that we used to have a Canadian in the physics department . . . and the good folks from up north don't use *conventional current* when teaching their students about circuits. They use *electron current*. Their diode symbol is the same as ours, but they assume that current *as they have defined it* moves *opposite* the direction of the arrow. And when dealing with the magnetic field set up by a current-carrying coil, they don't assume *conventional current* and use any of *the right-hand rules* to determine the field's direction, they assume *electron current* and use *the left-hand rule* to determine the field's direction.

f.) What's important to realize is that IT REALLY DOESN'T MATTER. As long as you are consistent when doing evaluations (you will understand this better after you have done some circuit problems), you will get the correct current magnitude (it's a scalar, remember) no matter *how* you visualize current movement in the circuit.

I will undoubtedly edge more toward *conventional currents* (that is what you will see in circuit diagrams unless told otherwise) because that is the way I was taught and that is the way your university courses will approach it, assuming you don't go to one of the few colleges in the U.S. that take the more exotic route. But as for this class, I will switch back and forth between the two.

Why? It's good for you. Who knows? You may go to college in Canada. And in any case, it will keep your brains working. That can never be a bad thing.

D.) Power:

1.) In an electric sense, *power* is defined as the amount of *work per unit time* an electrical element can provide or dissipate in a circuit.

2.) The unit for *power* is *joules/second*. This is given the special name *watts*.

3.) Although we will go into the math more completely later, for now all you need to know is that the voltage of, say, a power supply is *not* the only parameter that governs how much power the supply can provide to the circuit.

a.) The *power rating* P for a power supply is dependent not only on the supply's voltage V at a given instant, it is also dependent upon how much current i is being drawn from the supply at that point in time.

b.) Mathematically, the relationship is:

$$P = iV.$$

E.) AC Current:

1.) AC current *can't* be defined as the amount of charge that passes by a point per unit time. Why? The free charge-carriers in AC systems jiggle back and forth. The net flow over time is *zero*. So how do we deal with this?

2.) Take an AC power supply hooked across a light bulb, for instance.

a.) The power supply's voltage (remember, this is really a voltage *difference* across its terminals) rises and falls, then changes direction and rises and falls, etc. (Go back and look at Figure 13.4a.)

b.) The current through the light bulb likewise rises and falls, then changes direction and rises and falls, etc.

c.) In other words, a graph of the voltage across and current through the light bulb will show a maximum voltage difference V_o (i.e., the voltage amplitude as shown on the graph) and a maximum current i_o (i.e., the current's amplitude as shown on its graph) happening periodically.

i.) It will also show times when there is *no* voltage across the bulb or current flowing through the system.

d.) So how do we deal with this when it comes to the idea of current flow? We deal with it by asking the question, "What's important here?"

i.) What's important to this circuit is the amount of POWER being provided to the circuit by the power source, and the amount of POWER being dissipated by the light bulb (in an ideal situation, the two will be the same).

e.) The follow-up question is, "If we had a DC power supply, how much power would *it* have to provide to the circuit to match the power being provided by the AC supply?"

i.) The DC power provided to a DC circuit is $P = iV$, where i is the current being drawn from the DC power supply and V is the voltage across the DC power supply.

ii.) We should be able to use a similar expression to determine the amount of power an AC power supply provides to a circuit. It just isn't immediately obvious what voltage V and current i we should use to get *DC-equivalent* power value. After all, both the voltage and current are constantly changing in an AC circuit. It should be obvious that the values have to be fractions of the maximum current i_o and maximum voltage V_o. The question is, "*What* fraction?"

iii.) For reasons that will become clear later when you see the entire derivation, the *current value* used, called the RMS current, and the *voltage* value used, called the RMS voltage, are defined as

$$i_{RMS} = .707 i_o$$

and

$$V_{RMS} = .707\ V_o.$$

iv.) IMPORTANT: In other words, if you take .707 of the amplitude of both the AC current and the AC voltage functions, then multiply those RMS values together, you will come up with a number that designates the amount of DC power a DC power supply would have to provide to the circuit to run the light bulb in exactly the same way as does the AC source. See Figure 13.10.

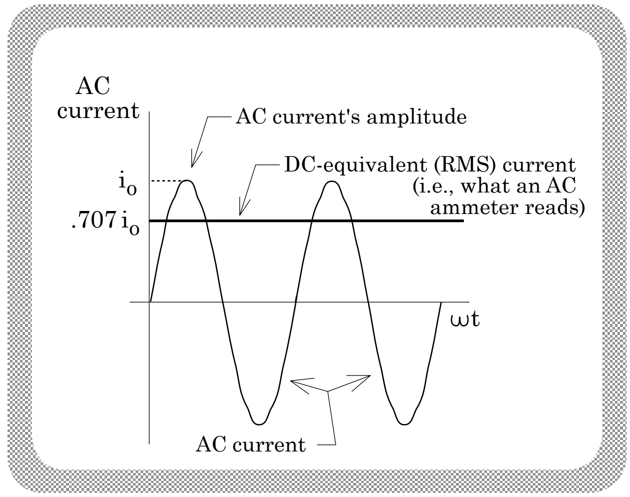


FIGURE 13.10

f.) An AC ammeter is not a polar device. There is no *high voltage* or *low voltage* terminal to it. What's more, the reading an AC ammeter gives is the *RMS value* for the current in the AC circuit. Similarly, the reading an AC voltmeter gives is the *RMS value* of the voltage across the element being checked.

F.) The Resistive Nature of Circuits:

1.) Consider a wire whose ends are attached to the terminals of a DC power supply. What limits the true flow of charge (i.e., the electron current) in the wire?

a.) When the wire is attached to the terminals, a DC voltage difference is set up between the ends and across the length of the wire. This almost instantaneously sets up a constant electric field in the wire. The protons in the wire feel the electric field but can't respond because they are fixed in the wire's atomic structure. The valence electrons, on the other hand, are free to respond and immediately begin to accelerate.

b.) As a given electron picks up speed, it sooner or later runs into an atom in the structure of the wire giving up its kinetic energy in the collision. The atom absorbing this energy will do so in one of several ways.

i.) Most of the absorbed energy will go into making the molecular structure vibrate more radically than usual. This shows itself as heat (this is why wires become hot when an electric current passes through them).

ii.) It is also possible for some of the absorbed energy to kick an electron in the atom's structure into a higher energy level. Once there, that electron will almost immediately begin to cascade down into lower and lower energy levels. As it moves on its way back down to its ground state, it dumps discrete bundles of energy with each jump (these bundles are called *photons*). If any of these bundles has the right energy content, your eyes will perceive them as light. (In fact, this is exactly how light is created at the atomic level.)

c.) Once an electron gives up its energy in a collision, it is once again accelerated in the electric field only to lose its accumulated kinetic energy with the next collision. This process occurs over and over again (see Figure 13.11).

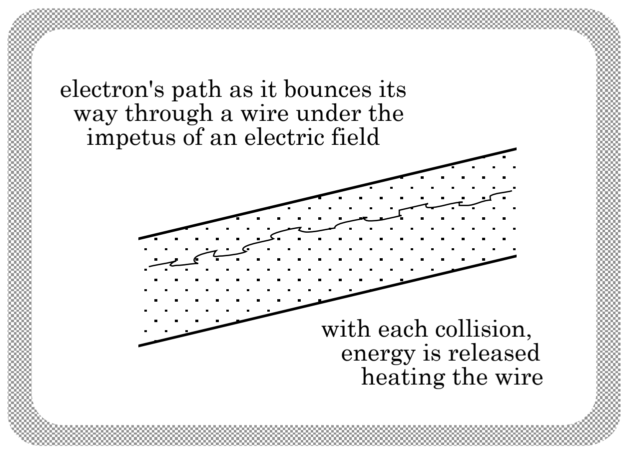


FIGURE 13.11

i.) As a minor side point, the average distance an electron travels before running into another atom in a given material is called *the mean free path*. This *mean free path* is a function of the density of the metal making up the wire.

d.) The fact that charge cannot move through a typical circuit unfettered, even if the circuit is only made up of a power supply and a wire, is related to what is called *the resistive nature* of circuits. When dealing with wire, this resistive nature is associated with what is called *resistance*.

2.) There are circuit elements that are designed specifically to provide resistance to a circuit.

a.) They are called *resistors*. Resistors are used to do two things.

i.) They limit the amount of current that can flow in a circuit. This is a reciprocal relationship. For a given voltage, a large resistor means you can expect a relatively small current, whereas a small resistor means you can expect a relatively large current.

ii.) Resistors convert electrical energy into heat energy (think toaster) or light energy (think light bulb) or mechanical energy (think electrical motor).

b.) It is easy to verify experimentally that if you place a voltage across a resistor and note its associated current, then double the voltage, the current will double. This is the equivalent of noting that the voltage across a resistor is directly proportional to the current through the resistor.

Note: There will be times when you will want to know if, in theory, a current will exist in a circuit in which resides at least one resistor and other circuit elements. One way to do this, remembering that the voltage across a resistor is proportional to the current through the resistor, is to see if there *is* a voltage across the resistor. If there is, a current must exist there. (This observation will make more sense later.)

c.) Called Ohm's Law, this proportionality between voltage and current is mathematically expressed in *equality* form as:

$$V = iR,$$

where V is the *voltage difference* across the element, i is the *current through* the element, and R is the proportionality constant.

d.) The proportionality constant R is the called *resistance*. Its units are *ohms*. The symbol for an *ohm* is Ω.

e.) Resistance values run anywhere from a few thousandths of an ohm (i.e., the resistance of a short piece of wire) to millions of ohms (i.e., an element that is manufactured to carry large resistance).

i.) 1000 ohms is abbreviated to 1 $k\Omega$, where the *k* stands for *kilo*, or thousand.

ii.) 1,000,000 ohms is abbreviated to 1 $M\Omega$, where the *M* stands for *mega*, or million.

Note: Even though a *43,500 ohm* resistor will usually be written as *43.5 kΩ* on circuit diagrams, all formulas using resistance values must be calculated in *ohms*. That is, you have to convert a resistance value like *43.5 kΩ* to *43,500 ohms* before you can use it in a problem.

f.) The circuit symbol for a resistor is shown in Figure 13.12.

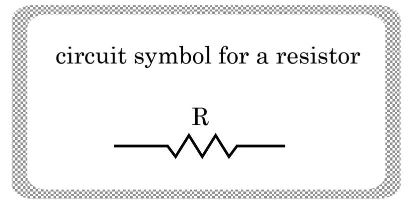


FIGURE 13.12

g.) Wires have very little net resistance to them.

i.) In fact, if you hook a wire to a power supply, the wire will get very hot very fast (high current produces lots of heat). This is why houses come complete with circuit breakers. When the current gets too large, the breaker trips and all current to the offending part of the system is turned off.

ii.) Wires have so little resistance associated with them that in circuit analysis problems, their resistance is ignored as negligible.

3.) Resistors act the same in AC circuits as they do in DC circuits.

4.) Bottom line: Voltage is not the only thing that governs how large a current will be in a circuit. There is also the resistive nature of the circuit. Although current flowing through the atomic matrix making up the wires of a circuit provides some small resistance to charge flow, the net resistance in a typical electrical circuit is assumed to be equal to the net resistance of the *resistors* in the circuit.

G.) Resistors (the movie):

1.) Resistors can show up in electrical circuits in lots of ways. There are instances when a circuit can be simplified by determining the "equivalent resistance" R_{eq} for the combination of resistors present. That is what we are about to examine.

2.) Figure 13.13 shows a *series* combination of resistors. *Series* combinations have several common characteristics.

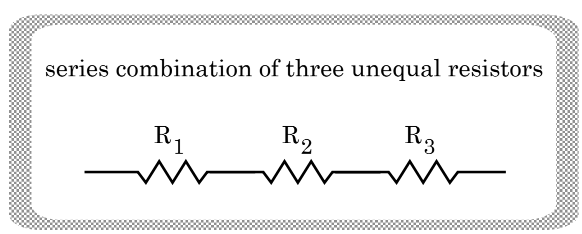


FIGURE 13.13

a.) Each resistor in a series combination is linked to its neighbor at one place only.

b.) Although the voltage across each element may differ, the *current* through each element will always be the same. Put another way, CURRENT is common to all elements in a series combination.

c.) There can be *no junctions* between elements in a series combination (if there were junctions, current through the different members would differ).

d.) Without proof, the equivalent resistance--the single resistor that can take the place of the series combination and still pull the same current--is:

$$R_{eq} = R_1 + R_2 + R_3 \ldots$$

3.) There are several observations that need to be made about series combinations. Specifically:

a.) R_{eq} in a series combination will always be LARGER than the largest resistor in the combination.

b.) Add a resistor to a series combination and R_{eq} will get LARGER. Remove a resistor from a series combination and R_{eq} will get SMALLER.

c.) Does this make sense on a conceptual level?

i.) Add a resistor to a series combination and, well, it adds resistance. This is exactly what our R_{eq} relationship suggests.

4.) Figure 13.14 shows a *parallel* combination of resistors. *Parallel* combinations have several common characteristics.

a.) Each resistor in a parallel combination is linked to its neighbor on both sides.

b.) Although the current through each element may differ (remember, current is dependent upon how large the resistance is and the voltage across the element), the *voltage across each element in parallel* will always be

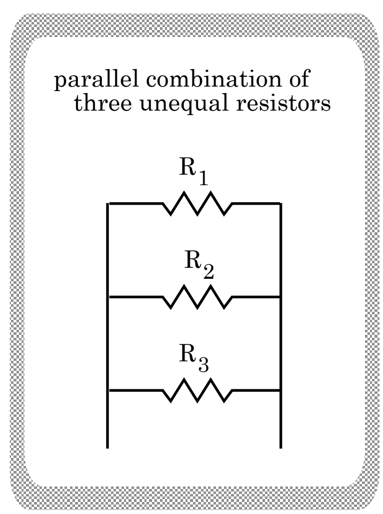


FIGURE 13.14

the same. Put another way, VOLTAGE is common to all elements in a parallel combination.

c.) There *will be* junctions between elements in a parallel combination. In fact, parallel combinations can be shown in several ways. (Figure 13.15 presents some examples.)

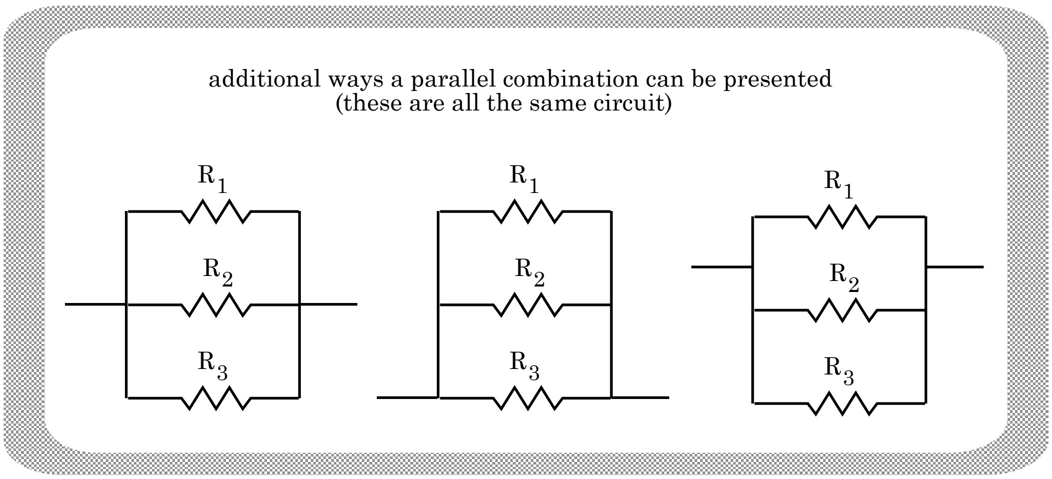


FIGURE 13.15

d.) Without proof, the equivalent resistance--the single resistor that can take the place of the parallel combination and still pull the same current--is

$$1/R_{eq} = 1/R_1 + 1/R_2 + 1/R_3 \ldots$$

5.) There are several observations that need to be made about parallel combinations. Specifically,

a.) R_{eq} in a parallel combination will always be SMALLER than the smallest resistor in the combination (try it with some numbers if this isn't obvious).

b.) Add an additional parallel resistor to a parallel combination and R_{eq} gets SMALLER (i.e., the current into the combination goes UP). Remove a resistor from a parallel combination and R_{eq} gets BIGGER (i.e., the current into the combination goes DOWN).

c.) Don't believe the statements made above? Try the math with two 10Ω resistors.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$= \frac{1}{(10\Omega)} + \frac{1}{(10\Omega)}$$

$$= \frac{2}{(10\Omega)}$$

$$\Rightarrow \quad R_{eq} = \frac{(10\Omega)}{2}$$

$$= 5\Omega.$$

i.) So far so good. R_{eq} is smaller than any of the resistors making up the combination.

d.) Now add another 10Ω resistor to the parallel combination. Doing so yields

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$= \frac{1}{(10\Omega)} + \frac{1}{(10\Omega)} + \frac{1}{(10\Omega)}$$

$$= \frac{3}{(10\Omega)}$$

$$\Rightarrow \quad R_{eq} = 3.3\Omega.$$

i.) It works. Adding an additional resistor to the parallel combination makes R_{eq} go DOWN.

e.) Does this make sense on a conceptual level?

i.) Think about it. According to Ohm's Law, what governs the AMOUNT OF CURRENT that flows through a given resistor is the size of the resistor AND the voltage difference across the resistor (i.e., $i = V/R$).

ii.) Assuming we have an ideal power supply (i.e., one that has no internal resistance--the significance of this will become evident shortly), does the voltage across any of the parallel resistors change when an additional resistor is added in parallel?

The answer to that is NO!

iii.) What does this mean? It means that with the addition of a new resistor in parallel, the amount of current passing through R_1 doesn't change, and the amount of current passing through R_2 doesn't change, and the amount of current passing through R_3 doesn't change, etc., but the amount of current being drawn from the power supply DOES change. Why? The supply now has to provide current to the newly added resistor.

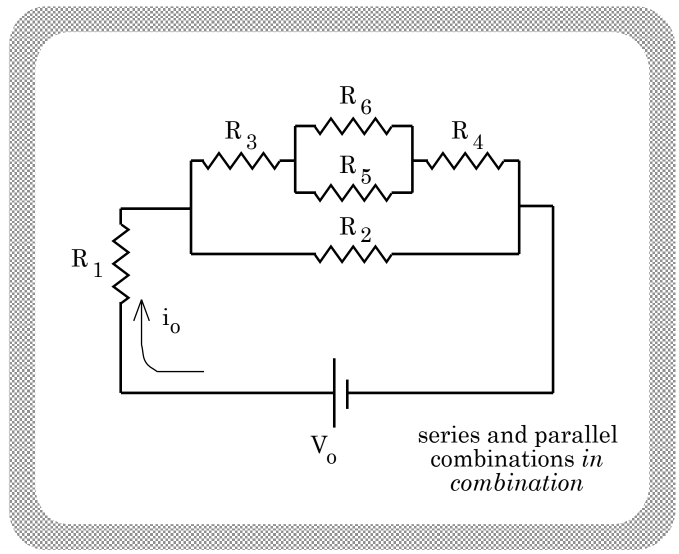


FIGURE 13.16

iv.) From the power supply's perspective, an increase in current suggests a decrease in effective resistance. That, as perverse as it may seem, is exactly what our parallel R_{eq} expression suggests. Add a resistor and R_{eq} goes down!

6.) What about series and parallel combinations in combination? Consider the circuit shown in the Figure 13.16.

a.) What are we looking at? As shown in Figure 13.17, this is a *series combination*--R_1 in series with a mess of resistors (mess #1 in Figure 13.17). The mess is ac-

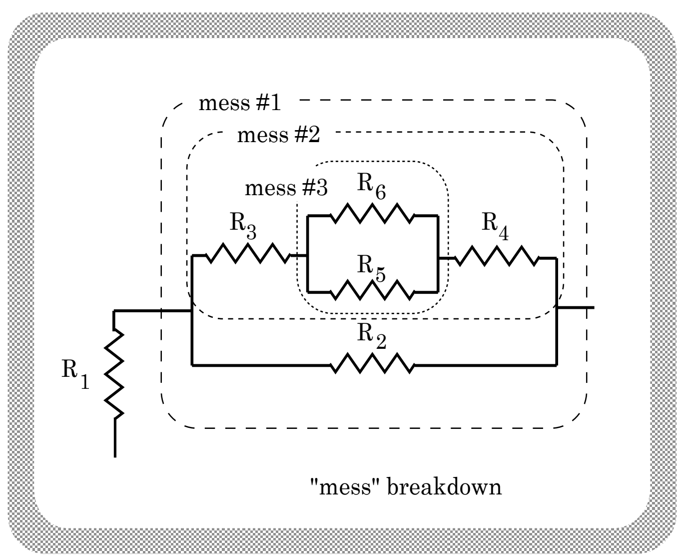


FIGURE 13.17

tually a parallel combination--R_2 in parallel with mess #2. Mess #2 is really R_3 and R_4 in series with the parallel combination of R_5 and R_6.

b.) The progression of untanglement is shown in Figure 13.18.

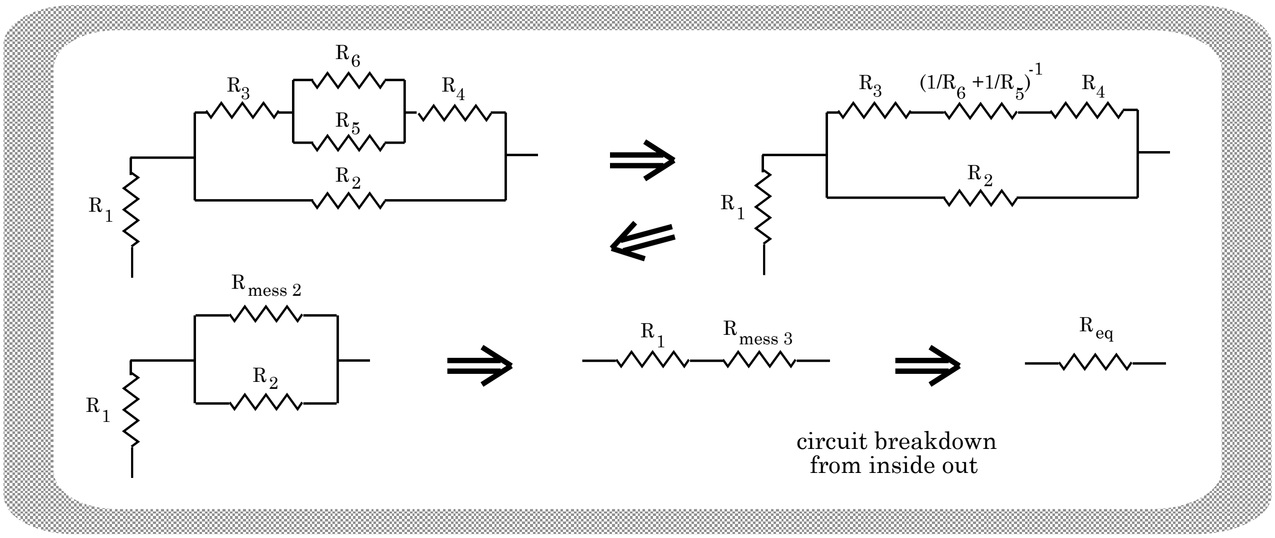


FIGURE 13.18

Starting from the *inside going outward*, *mess #3*--R_5 and R_6 in parallel--has an equivalent resistance of:

$$R_{mess\ 3} = (1/R_5 + 1/R_6)^{-1}.$$

c.) The equivalent resistance for *mess #2*:

$$R_{mess\ 2} = R_3 + R_4 + R_{mess\ 3}.$$

d.) The equivalent resistance for *mess #1* is:

$$R_{mess\ 1} = [1/R_2+1/(R_{mess\ 2})]^{-1}.$$

e.) The *net* equivalent resistance for the entire combination after the "mess" expressions are inserted is:

$$\begin{aligned} R_{eq} &= R_1 + R_{mess\ 1} \\ &= R_1 + \{1/R_2 + 1/[R_3 + R_4 + (1/R_5 + 1/R_6)^{-1}]\}^{-1}. \end{aligned}$$

Note: Questions like this are a lot easier if you use numbers from the beginning. The problem with this lies in the fact that once we begin combining numbers, quantities get absorbed into other quantities and it becomes very difficult for a second reader to follow one's reasoning.

Solution to the dilemma: Use sketches like those shown in Figures 13.18 and 13.19.

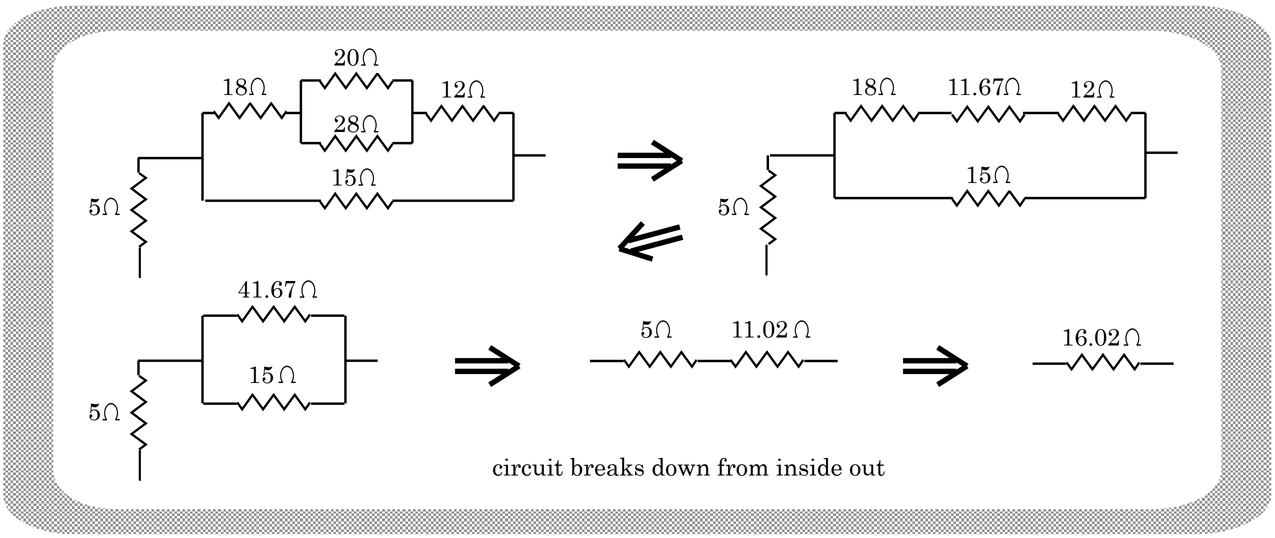


FIGURE 13.19

7.) The problem outlined in #6, but with numbers:

a.) Let resistors $R_1 = 5\ \Omega$, $R_2 = 15\ \Omega$, $R_3 = 18\ \Omega$, $R_4 = 12\ \Omega$, $R_5 = 28\ \Omega$, $R_6 = 20\ \Omega$. Determine the current drawn from a 12 volt battery as shown in Figure 13.16.

b.) The progression of equivalent resistances is shown in Figure 13.19.

c.) This is how you deal with series and parallel combinations in combination.

H.) A Few More Words About Power:

1.) If you will remember, *power* was defined as the amount of *work per unit time* a circuit element can either *provide* or *dissipate* in a circuit. That is, it is related to the *time rate of work* done on charge carriers moving through a circuit. As would be expected, the units of power are *joules per second* or *watts*.

2.) <u>Bottom line</u>:

a.) Knowing the *power rating* of an element tells you how much *energy per unit time* the element can generate or dissipate, depending upon the element's function.

b.) If the element is a *DC power supply*, energy is *supplied* to the system and the power rating is iV, where V is the *voltage across the terminals* and i is the *current drawn from* the power source.

c.) If the element is an *AC power supply*, energy is *supplied* to the system and the power rating is iV, where V is the *RMS voltage* associated with the supply and i is the *RMS current drawn from* the power source.

d.) If the element is a *resistor*, energy is *removed from the system* as heat or light or converted to mechanical energy.

i.) For resistors, the DC power rating is iV, i^2R, or V^2/R depending upon which variables you know.

ii.) These relationships come from using the expression $P = iV$ in conjunction with Ohm's Law, or $V = iR$).

iii.) As above, *RMS* values are used when the current in the circuit is AC.

I.) Thinking Conceptually About Circuits:

1.) This section is a kind of pre-test devoted to the conceptual side of circuit analysis. It is designed to help you THINK, not to give you stuff to memorize. Don't shortchange the process. Try to figure out each question before turning to the solutions (they are in the next section).

2.) A large resistor R and a small resistor $r = R/2$ are placed in series across an ideal power supply. Use the lettered points shown in Figure 13.20 and answer the following questions.

a.) If you switched the resistors so that r is first and R second, how will the current through r change?

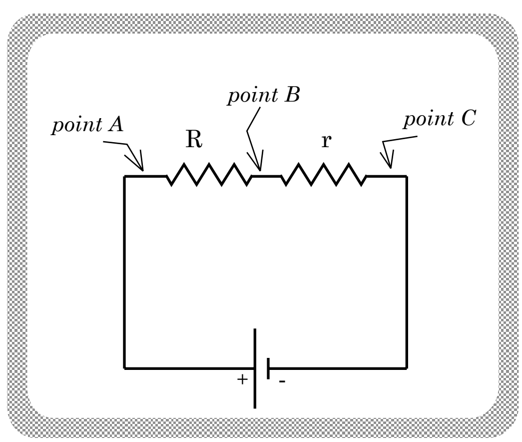


FIGURE 13.20

b.) Where is the current the greatest, at *Point A*, *Point B*, or *Point C*?

c.) Is the voltage greatest across the larger or smaller resistor?

d.) If you put another resistor of size *R* in parallel with the original *R* (see Figure 13.21):

i.) How will the current in the original *R* change? That is, will it go up, stay the same, or go down?

ii.) How will the current in *r* change?

iii.) If these are ideal light bulbs, which bulb (or bulbs) will dissipate the most power?

iv.) How will the voltage at *Point B* change from the original situation?

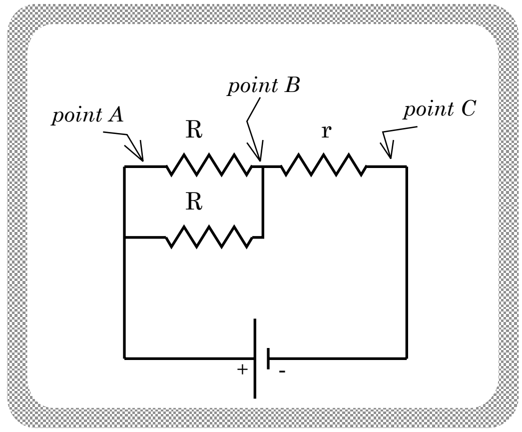


FIGURE 13.21

v.) How will the voltage at *Point C* change from the original situation?

3.) A combination of switches and identical light bulbs is placed across an ideal power supply as shown in Figure 13.22.

a.) Switch #1 is closed as shown in the sketch. Rank the light bulbs in order of brightness, starting with the brightest. Be able to explain your response.

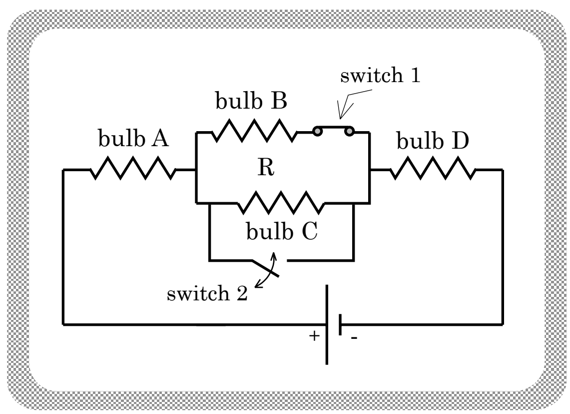


FIGURE 13.22

b.) Switch #1 is opened. Rank the light bulbs by brightness.

c.) With Switch #1 closed, Switch #2 across bulb C is closed. Rank the light bulbs in order of brightness.

4.) All of the light bulbs are the same size in the three circuits shown in Figure 13.23. Of all the bulbs shown, which will burn the brightest?

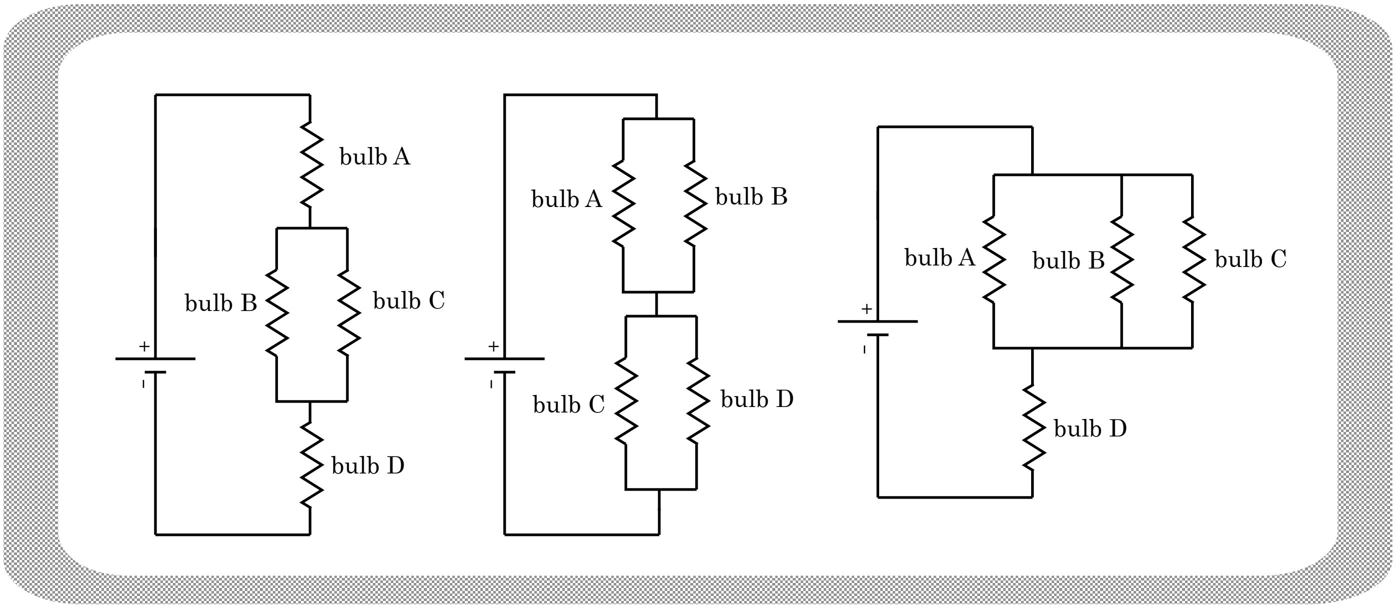


FIGURE 13.23

5.) A 100 watt light bulb is placed in series with a 15 watt light bulb. The two are then placed across an ideal power supply. Which light bulb will burn the brightest?

J.) Solutions to Conceptual Questions in Section I:

1.) The previous section had questions that were designed to provoke you into thinking about the differences between current and voltage and the relationships that exist between those quantities in the context of resistor-laden DC circuits. The following is a summary of the things you hopefully figured out on your own.

2.) A large resistor R and a small resistor $r = R/2$ are placed in series across an ideal power supply. Use the lettered points shown in Figure 13.20 and answer the following questions.

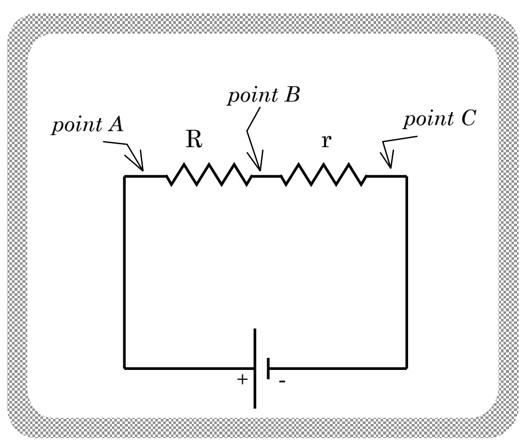


FIGURE 13.20

a.) If you switched the resistors so that *r* is first and *R* second, how will the current through *r* change?

Ans.) The current through a series combination is dependent upon the voltage impressed across the branch and the total resistance associated with the branch. Switching the order of resistors does nothing to either of these quantities, so the current should stay the same.

b.) Where is the current the greatest, at *Point A*, *Point B*, or *Point C*?

Ans.) The current is the same at every point in a series combination.

c.) Is the voltage greatest across the larger or smaller resistor?

Ans.) The voltage across a resistor is proportional to the current through the resistor and the size of the resistor. As the *current* through a series combination is the same for each element, the larger voltage drop in the combination will happen across the larger resistor, or *R* in this case.

d.) If you put another resistor of size *R* in parallel with the original *R* (see Figure 13.21):

i.) How will the current in the original *R* change? That is, will it go up, stay the same, or go down?

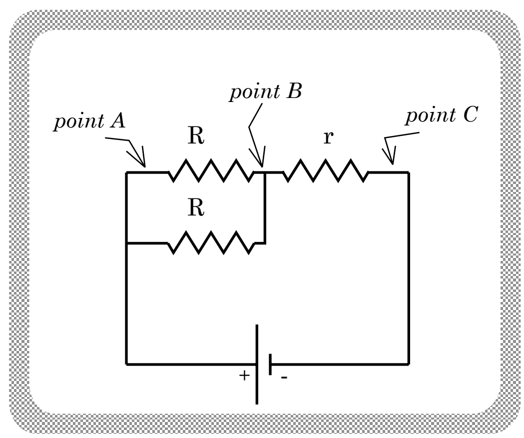


FIGURE 13.21

Ans.) By putting another resistor in the circuit, you change the current being drawn from the power supply. By having the added resistor be in *parallel* to one of the original resistors, you effectively *drop* the equivalent resistance of the overall circuit and *increase* the current being drawn from the battery (remember, adding parallel resistors means there is one more branch through which current must pass, hence the required increase in current from the power supply). Given the way the circuit is set up, *all that current* will go through *r*. This means the *voltage* across *r* goes up. As the total voltage across *r* and the parallel combination must stay the same (it is just the power supply voltage), that means the *voltage* across *R* (and its parallel partner) must go DOWN. If the voltage across *R* goes down, THE CURRENT THROUGH *R* MUST GO DOWN. (And no, this wasn't obvious . . . this was a nasty question.)

ii.) How will the current in *r* change?

Ans.) As outlined above, the current through *r* will increase.

iii.) If these are ideal light bulbs, which bulb (or bulbs) will dissipate the most power?

Ans.) Power is determined using i^2R. Because the two resistors in parallel are the same size, each will experience the same current flow. Because those two streams of current will come together, then flow through r, r will evidently have *twice the current* through it than does either of the two individual R's. On the other hand, R is twice as large as r. So which resistor is going to have the larger i_2r power rating?

If we let i be the current through one R, and let $R = 2r$, we can use our power expression on one of the R's and get $P_R = i^2(2r) = 2i^2r$. Doing the same for r yields $P_r = (2i)^2r = 4i^2r$. The net result is that r will dissipate more power.

iv.) How will the voltage at *Point B* change from the original situation?

Ans.) We have already established that the voltage drop across r increases, so the voltage at B must increase, also.

v.) How will the voltage at *Point C* change from the original situation?

Ans.) The voltage at C is the same as the voltage of the ground terminal of the battery. That voltage will not change in any case.

3.) A combination of switches and identical light bulbs is placed across an ideal power supply as shown in Figure 13.22.

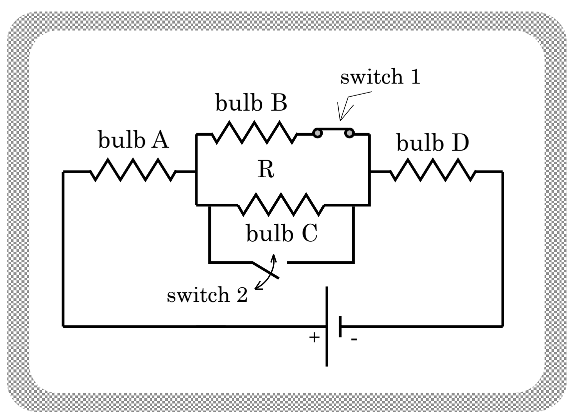


FIGURE 13.22

a.) The top switch is closed while the bottom switch is open (see the sketch). Rank the light bulbs in order of brightness, starting with the brightest. Be able to explain your response.

Ans.) The power supply will provide some net current i_o to the circuit. ALL of that current will go through bulbs A and D, so they will be the brightest. (Being in parallel, only half of the current will go through bulbs B and C--they will be equally bright but not as bright as the other bulbs.)

Note: It probably isn't obvious, but twice the current will not produce twice the brightness. Brightness is not a *linear* function of current.

b.) The top switch is opened. Rank the light bulbs by brightness.

Ans.) With the top switch open, bulb *B* is completely removed from the circuit. That means all the current will pass through all the remaining bulbs and they will all have equal brightness.

Note: With the removal of one parallel resistor, the equivalent resistance of the circuit goes *up* and the net current goes *down*. That means the brightness that is observed in the bulbs of this circuit will be *less than* the brightest of the bulbs in the first situation.

c.) With the top switch closed, the switch across bulb C is closed. Rank the light bulbs in order of brightness.

Ans.) In this case, there is a path for current to take around bulb *C* and an additional path for current to take around bulb *B*. That means the net current in the circuit will all go through bulbs *A* and *D*--they will be equally bright--and almost no current will go through bulbs *B* and *C*.

Note: How will the brightness of *A* and *D* compare to the brightness of the bulbs in the original situation? The wire around bulbs *B* and *C* has VERY LITTLE CURRENT. As the resistance of a parallel combination of resistors is smaller than the smallest resistor in the bunch, the effective resistance of that combination is zero. That means the only resistance that limits current in the circuit is the resistance associated with bulbs *A* and *D*. In turn, that means more current will flow in the circuit and the bulbs that do shine will shine more brightly than the bulbs in the original configuration.

4.) All of the light bulbs are the same size in the three circuits shown in Figure 13.23 (next page). Of all the bulbs shown, which will burn the brightest? (Look at the sketch and try to figure it out before reading the answer below.)

Ans.) The bulb that shines brightest will be the bulb with the greatest current through it.

To figure this out, if the resistance of each bulb is R, the net resistance of the *parallel combination* in the first circuit will be $R/2$ and the net resistance for that entire *first circuit* will be $2.5R$. The power supply current in that circuit will be $V_o/2.5R$.

The two parallel resistor combinations in the *second circuit* will EACH have an equivalent resistance of $R/2$ for a total resistance of R. The power supply current in that circuit will be V_o/R.

The *three parallel resistor combination* in the *third circuit* will have an equivalent resistance of $R/3$ for a *total resistance* or $1.33R$. The power supply current in that circuit will be $V_o/1.33R$.

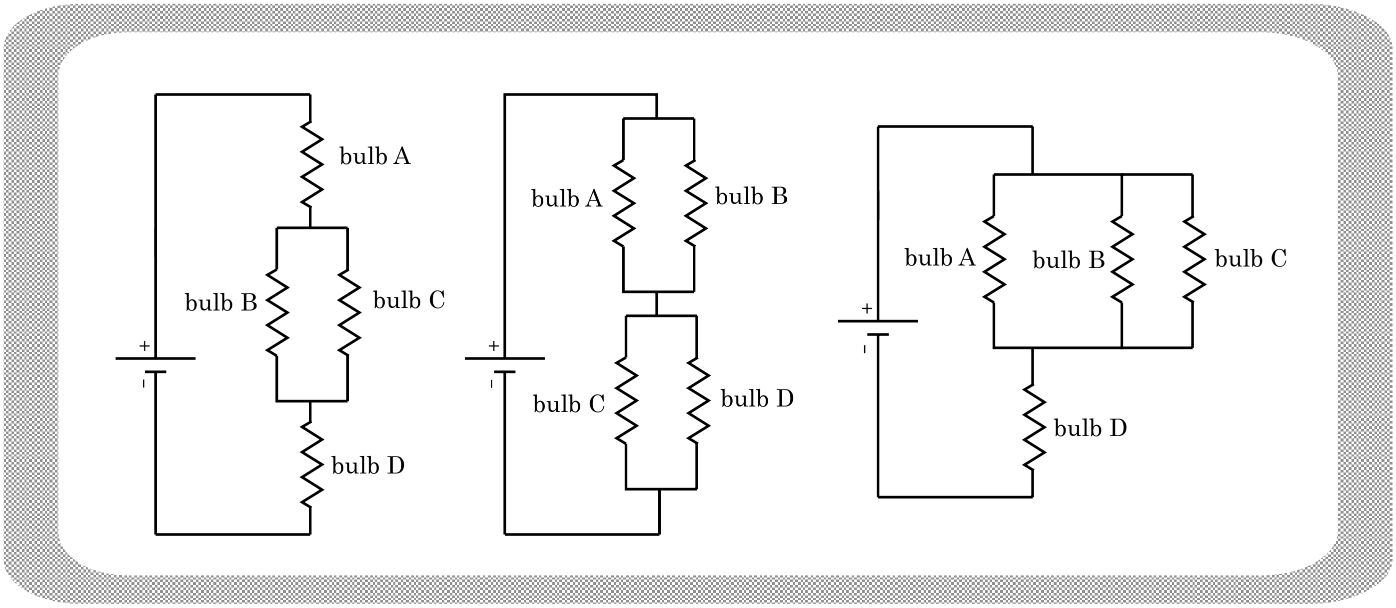


FIGURE 13.23

The temptation is to assume that because the current in the second circuit will be greatest, the second circuit is the answer. Unfortunately, only *half* of the current drawn from the power supply in that circuit will go through each resistor (the current will split when it enters the parallel sections leaving each resistor with current $V_o/2R$). In fact, the resistor that will have the greatest current flowing through it will be the single resistor found in the third circuit.

5.) A 100 watt light bulb is placed in series with a 15 watt light bulb. The two are then placed across an ideal power supply. Which light bulb will burn the brightest?

Ans.) The expression $P = i^2R$ suggests that high power is the consequence of high current (it is the current variable that is squared in the relationship). For a fixed voltage, as R goes down, i goes up. That means a 100 watt bulb (with its high power need) must have high current and, hence, low resistance. Conversely, the 15 watt bulb must need less current and, as a consequence, must have more resistance associated with it.

K.) DC Current Revisited–Determining Branch Currents By the Seat of Your Pants:

1.) It is now time to see how much of this you can apply to circuit analysis at a conceptual level. The problems you find below require a fair amount of creative thinking. THIS IS SOMETHING YOU NEED TO DEVELOP. At the very least, you should *intellectually* understand what is being done here even if there is no bottom-line approach being presented.

2.) At this point, we haven't much to work with. We know how to determine the *equivalent resistance* of a combination of resistors, we know that the voltage across a resistor R through which i*'s* worth of current flows is iR, and we know the obvious--that the amount of current that flows into a node (i.e., a junction) must equal the amount of current that flows out of the node. Armed with these facts, let's see what we can do with a circuit analysis problem.

3.) Consider the circuit shown in Figure 13.24. Assume $R_1 = 25\ \Omega$, $R_2 = 18\ \Omega$, $R_3 = 23\ \Omega$, and $V_o = 15$ *volts*. What can we deduce about the circuit?

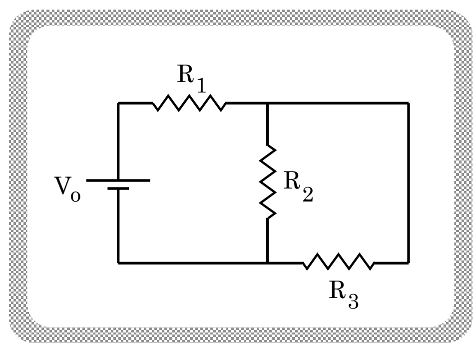


FIGURE 13.24

a.) Things to notice:

i.) There are *three branches* and *two nodes* in the circuit.

ii.) There is one simple *parallel* combination of resistors in the circuit (whether it is obvious or not, R_2 is in parallel with R_3--see "Big Note" below), and R_1 is in *series* with that parallel combination.

Big Note: Circuits are often drawn with the convenience of the designer overshadowing the convenience of all others. Following that tradition, notice that in Figure 13.24, R_3 is in parallel with R_2 even though the two do not look like a standard parallel set-up. This can be easily remedied by sliding R_3 around to the position shown in Figure 13.25.

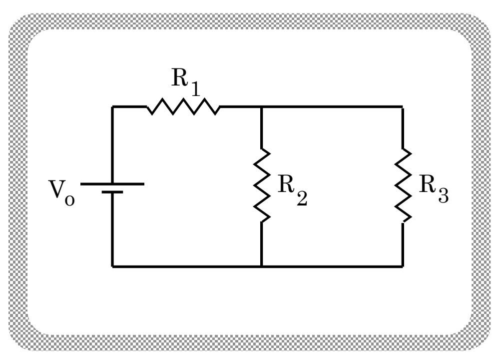


FIGURE 13.25

The moral: Look at your circuits closely. Be sure there are no oddball sections that can be simplified by using your head. Do not be bashful about redrawing your circuit if you think the circuit creator is being tricky or obscure.

b.) The first thing we need to do is to define variables for each branch's current. Figure 13.26 does this. Note that the *i* terms are *conventional currents.*

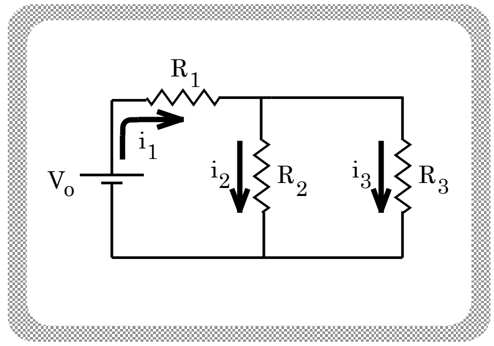


FIGURE 13.26

c.) If we could determine R_{eq} for the entire circuit, we could use that information to calculate the current that must be drawn from the power supply (i.e., i_1) using $i_1 = V_o/R_{eq}$.

That is where we will start (see Figure 13.27).

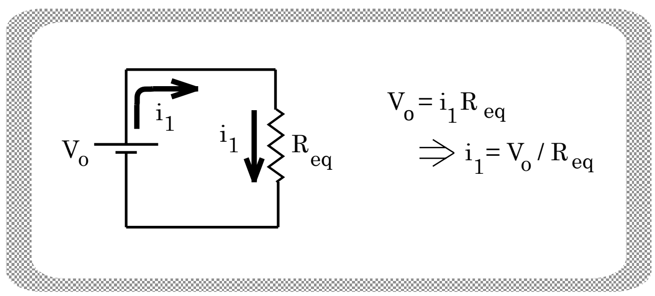


FIGURE 13.27

d.) R_1 is in series with the parallel combination of R_2 and R_3. Mathematically, the equivalent resistance for this combination is:

$$\begin{aligned} R_{eq} &= R_1 + (1/R_2 + 1/R_3)^{-1} \\ &= (25\ \Omega) + [1/(18\ \Omega) + 1/(23\ \Omega)]^{-1} \\ &= 35.1\ \Omega. \end{aligned}$$

e.) Using the R_{eq} and the known power supply voltage V_o, we get:

$$\begin{aligned} i_1 &= V_o/R_{eq} \\ &= (15 \text{ volts})/(35.1\ \Omega) \\ &= .427 \text{ amps}. \end{aligned}$$

f.) We now need to go back to the original circuit and see what else we can deduce. One thing we know is that with i_1, we can use Ohm's Law to determine the voltage drop across R_1. That is:

$V_1 = i_1 R_1$
$= (.427 \text{ amps}) (25\ \Omega)$
$= 10.675 \text{ volts.}$

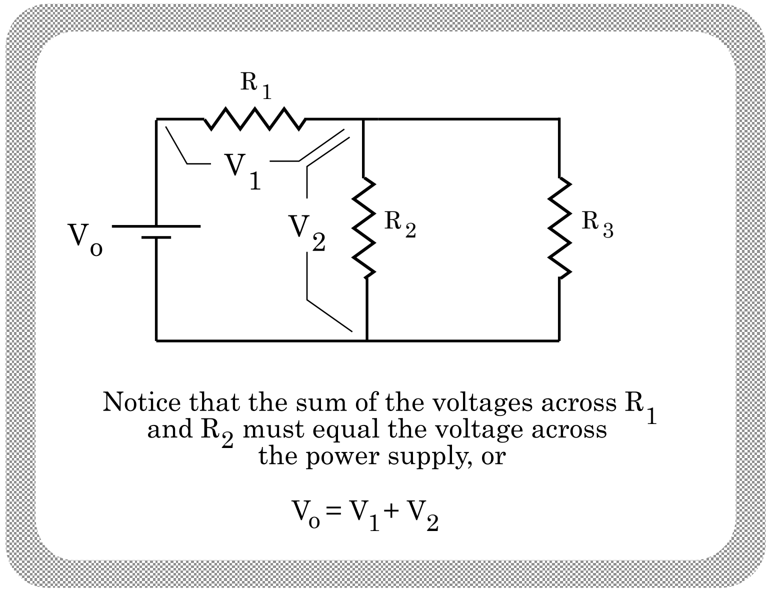


FIGURE 13.28a

g.) So far so good. What else do we know? We know the voltage difference across R_1 added to the voltage difference across R_2 must equal the voltage difference V_o across the battery (see Figure 13.28a). If we take the voltage across R_1 to be V_1 (remember, we already know this) and the voltage across R_2 to be V_2, we can write:

$$\begin{aligned} V_o &= V_1 + V_2 \\ \Rightarrow \quad V_2 &= V_o - V_1 \\ &= (15 \text{ volts}) - (10.675 \text{ volts}) \\ &= 4.325 \text{ volts.} \end{aligned}$$

h.) Because R_2 and R_3 are in parallel, V_2 is the voltage across both R_2 AND R_3 (see Figure 13.28b). Using that information and Ohm's Law, we get:

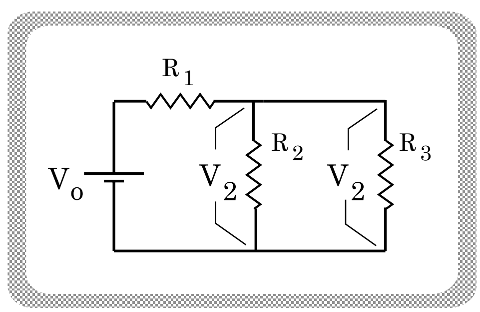


FIGURE 13.28b

$$\begin{aligned} i_2 &= V_2/R_2 \\ &= (4.325 \text{ volts})/(18\ \Omega) \\ &= .24 \text{ amps.} \end{aligned}$$

$$\begin{aligned} i_3 &= V_2/R_3 \\ &= (4.325 \text{ volts}) / (23\ \Omega) \\ &= .188 \text{ amps.} \end{aligned}$$

Big Note: The current *into* the *upper node* (i.e., the upper junction) is i_1 whereas the current *out of* the *upper node* is $i_2 + i_3$. It should not be surprising to find that to a very good approximation (giving the round-off error, etc.):

$$i_1 = i_2 + i_3,$$

or

$$(.427 \text{ amps}) = (.24 \text{ amps}) + (.188 \text{ amps}).$$

4.) So what have we done in this section? We have successfully analyzed a circuit problem using nothing more than Ohm's Law, the idea of *equivalent resistance*, and a little bit of logic. There is a considerably more formal way to do a problem like this, but learning how to tackle circuits using this *seat of the pants* approach is really more useful for our needs. There are, though, two more tools we could use. They are embodied in what are called *Kirchoff's Laws*.

L.) Kirchoff's Laws–Preliminary Definitions and Discussion:

1.) Kirchoff's First Law states that the *net* (total) *current into a node* (i.e., a junction) must equal the *net current out of a node.*

a.) The observation made in the *Big Note* in the previous section numerically substantiates this claim. The calculated value (rounded) for the current *entering* the *top node* in that problem was $i_1 = .43$ *amps*; the calculated values for the currents *leaving* that node were $i_2 = .24$ *amps* and $i_3 = .19$ *amps*. The sum of the "currents in" equals the sum of the "currents out."

2.) Kirchoff's Second Law states that the sum of the *voltage differences* around any closed loop must equal *zero*.

a.) Consider the circuit in Figure 13.29.

b.) Ohm's Law suggests that the current

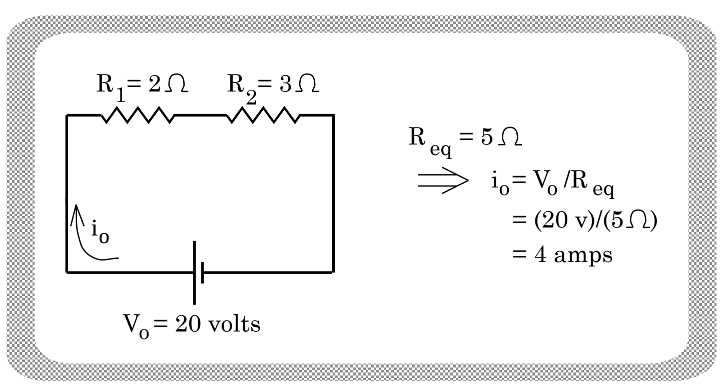


FIGURE 13.29

drawn from the power supply will be 4 amps.

c.) Figure 13.30 summarizes the absolute voltages of various points around the circuit. What is the justification for those values?

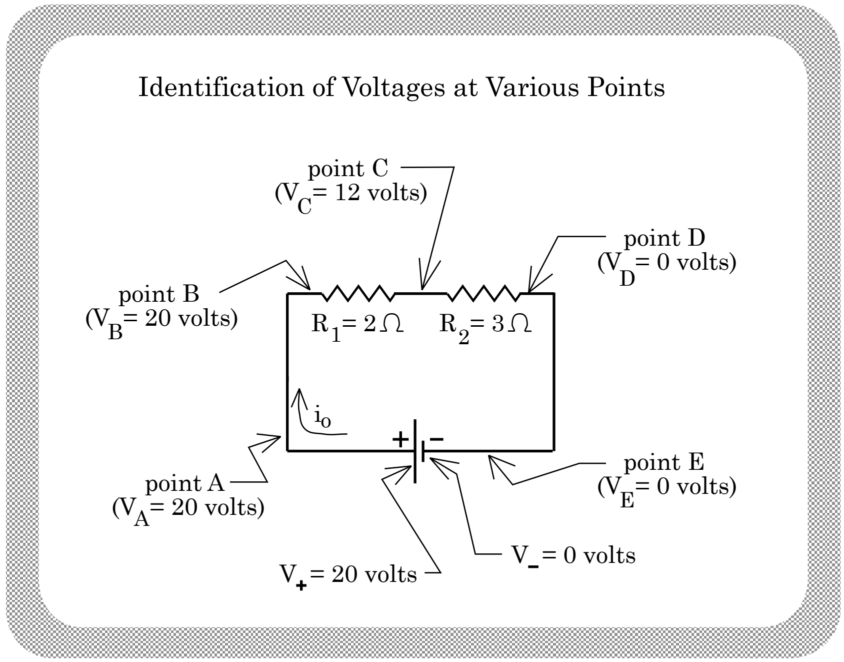


FIGURE 13.30

i.) The voltage at the high voltage side of the battery (i.e., the + terminal) can be assumed to be 20 volts if the power supply is rated as *20 volts*.

ii.) Think back. The absolute electrical potential at a point (i.e., the voltage at that point) measures the amount of *potential energy per unit charge* that is available at the point. In other words, voltage is a modified *energy* quantity.

iii.) Very little energy loss occurs as one moves through wire. That means the *potential energy per unit charge* does not change much between two points separated only by wire. That means the *voltage drop* between those two points will be negligible. This, in turn, means that the power supply's *positive plate* and point labeled *A* will have the same voltage, or 20 volts.

iv.) Because there is nothing but wire between *point A* and *point B*, there will be only a negligible voltage drop between those two points also. That means the voltage at *point B* will essentially be the same as at *point A*, or 20 volts.

v.) Conventional current (i.e., *i* as defined in the sketch) flows from *higher voltage* to *lower voltage*. That means the left side of resistor R_1 must be at higher voltage than the right side. It also means there must be a *voltage drop* as one traverses from left to right across the resistor.

vi.) The magnitude of the voltage drop across R_1 will be

$$
\begin{aligned}
V_1 &= i_o R_1 \\
&= (4 \text{ amps})(2 \text{ ohms}) \\
&= 8 \text{ volts.}
\end{aligned}
$$

vii.) As the VOLTAGE CHANGE across R_1 is 8 volts, and as *point B* is 20 volts, *point C* must be 12 volts.

viii.) There is nothing but wire between the right side of R_1 and the left side of R_2, so the voltage change between those two points will be negligible.

ix.) The magnitude of the voltage drop across R_2 will be

$$
\begin{aligned}
V_2 &= i_o R_2 \\
&= (4 \text{ amps})(3 \text{ ohms}) \\
&= 12 \text{ volts.}
\end{aligned}
$$

x.) As the VOLTAGE CHANGE across R_2 is 12 volts, and as *point C* is 12 volts, *point D* must be zero volts (plus a hair, which we will ignore).

xi.) Because there is nothing but wire between *point D* and *point E*, there will be only a negligible voltage drop between those two points and the voltage at *point E* will essentially be the same as at *point D*, or zero volts.

xii.) There is nothing but wire between *point E* and the ground side of the power supply. It shouldn't be surprising that *point E's* zero volts matches up with the *zero volt* rating of the negative terminal of the power supply.

xiii.) Notice that if we started at *point E* and registered the VOLTAGE CHANGES as we traversed clockwise around the circuit, we would find a *voltage increase* across the battery of 20 volts, a *voltage decrease* across R_1 of 8 volts, and a voltage *decrease* across R_2 of 12 volts. Adding up all of those voltage changes (see Figure 13.31) would yield

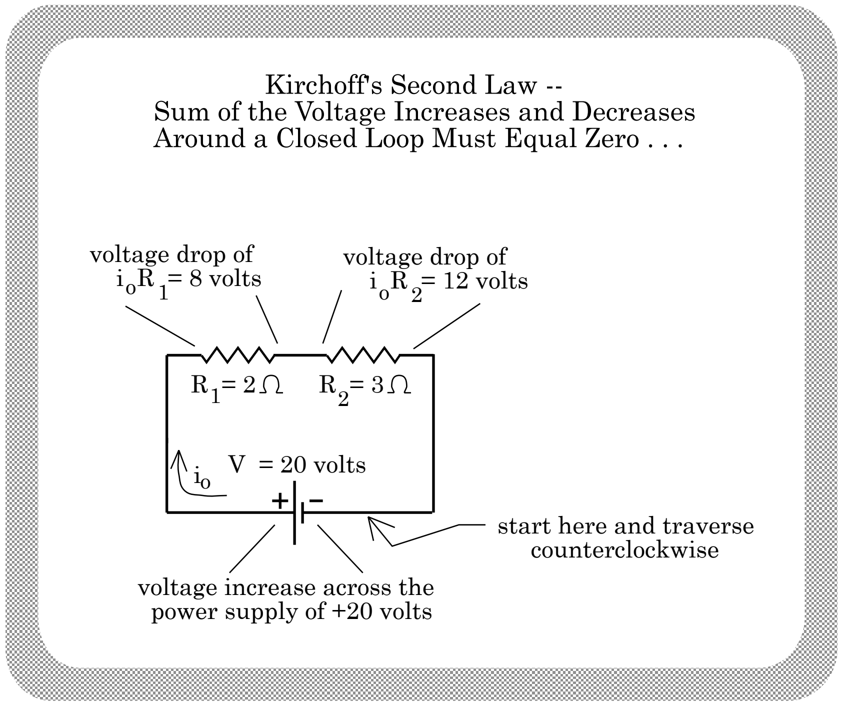


FIGURE 13.31

"the change between *point E* and *point E*" = +20 volts - 8 volts - 12 volts
= 0 volts.

OH, HOW CLEVER WE ARE! WE HAVE JUST DEDUCED THAT IF WE TRAVERSE AROUND A CLOSED PATH, THE NET *VOLTAGE CHANGE* BETWEEN A POINT AND ITSELF WILL BE . . . ZERO!!!

xiv.) This is Kirchoff's Second Law. Traverse around any closed path in a circuit and the sum of the voltage changes around that path will be zero.

d.) When doing circuit problems, we don't normally have numbers to work with as was the case in the preceding problem. For most cases, the voltage changes have to be taken care of using algebraic symbols. In fact, this is a considerably more useful way to approach things. If we had done the above problem from that perspective, the following is what it would have looked like.

i.) Traversing clockwise, the voltage change across the power supply would be $+V_o$, where the *positive sign* would denote that the voltage *increased* as we traversed counterclockwise from the start-point *through* the power supply.

ii.) The voltage change across R_1 would be $-i_oR_1$, where the negative sign would denote a voltage *decrease*. (How so? If you traverse in the direction of the conventional current, you will be moving from *higher* to *lower* voltage. This is a voltage DECREASE.)

iii.) The voltage change across R_2 would be $-i_oR_2$, where the negative sign would denote a voltage *decrease*.

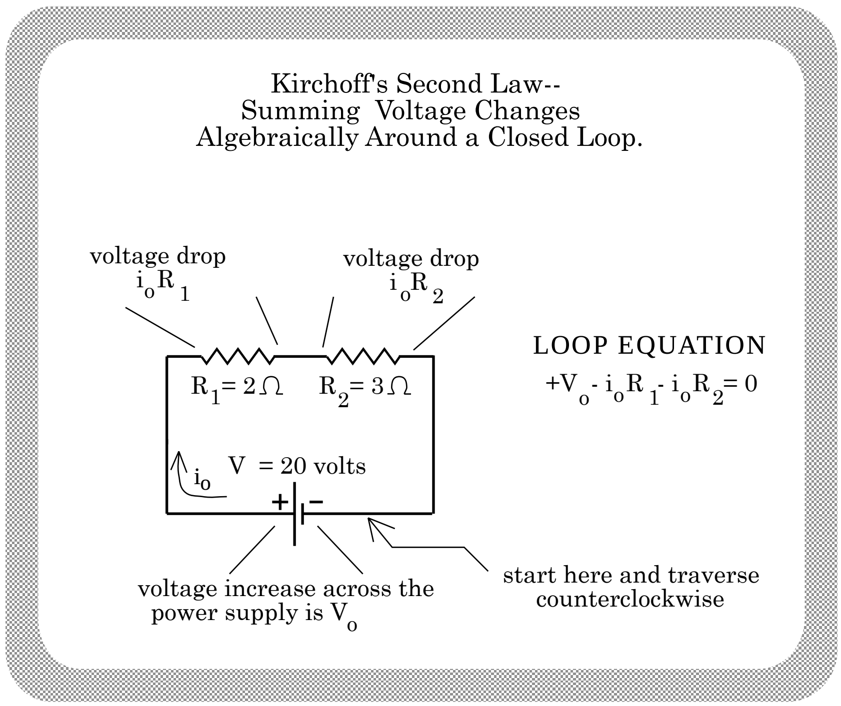


FIGURE 13.32

iv.) Putting it all together in what is commonly called a *loop equation* (see Figure 13.32), we would write:

$$V_o - i_oR_1 - i_oR_2 = 0.$$

THIS IS THE FORM YOU WILL USUALLY USE.

4.) Kirchoff's Laws--The Technique: There is a specific technique to using Kirchoff's Laws. Simply stated, that technique is described below (it will be applied to a problem in the next section).

a.) Begin by defining and appropriately labeling a *current* for every branch (a branch starts and ends at a junction). We will assume that all currents are *conventional*.

Note 1: Students often assume one has to "psyche out" a circuit problem before actually doing it. That is, you might think it is important to somehow determine the *direction* of the current in a particular branch before assigning a *current variable* to that branch. The beauty of this technique is that *it does not matter* whether the current direction is obvious or not. As long as you are consistent throughout the problem, the mathematics will take care of any incorrectly assigned current directions. It will do so by generating a negative sign in front of each calculated current value whose direction was incorrectly assumed.

THIS IS IMPORTANT. A negative sign in front of a current value doesn't mean anything is discharging or the current is decreasing. It means only that you have assumed the WRONG DIRECTION of current flow for that particular situation. It is no big deal--the numerical solution will still tell you how many coulombs of charge are passing a given point in the branch per unit time. It is just that a negative sign MEANS SOMETHING. You should know what that "something" is.

Note 2: If this is not clear, don't worry. The current defined as i_3 in the problem presented in the next section has intentionally been defined in the wrong direction. When you get there, watch to see how the math takes care of the oversight.

b.) Write out *Node Equations*: Pick a node and apply *Kirchoff's First Law* for that junction. Do this for as many nodes as you can find, assuming you aren't duplicating equations (you will see just such a duplication situation in the next section's problem).

c.) Write out *Loop Equations*: Choose a closed loop and apply *Kirchoff's Second Law* to that closed path. Do so for as many loops as are needed to accommodate the number of unknowns you have.

d.) Solve the Loop and Node Equations *simultaneously* for the currents in the circuit.

Note 3: There is a technique for solving *selected* currents using matrix analysis. There is also a clever way to use your calculator to solve for ALL the unknown currents. Both are summarized later in the book. For now, though, you should concentrate on the seat of the pants approach to analyzing circuits.

M.) More Circuits Analysis:

1.) There is a very formal way to approach circuit analysis that is intimately related to Kirchoff's Laws, but there are many instances when short-cutting the Kirchoff process is definitely the easiest way to go. To see how these shortcuts work, we will analyze the circuit shown in Figure 13.33 for a number of different initial conditions. In doing so, you will see all the ways to proceed.

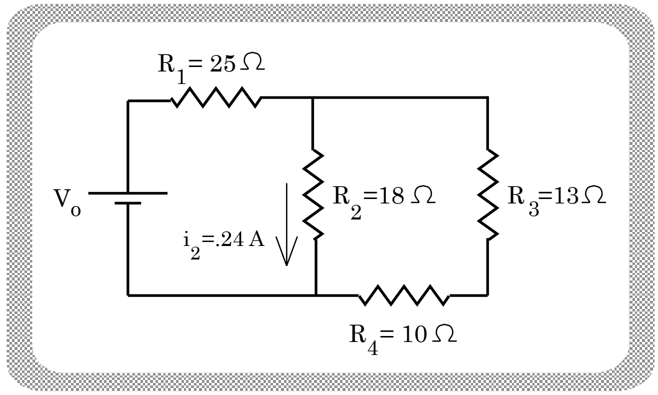


FIGURE 13.33

2.) Look at Figure 13.33. Assume that $R_1 = 25\ \Omega$, $R_2 = 18\ \Omega$, $R_3 = 13\ \Omega$, $R_4 = 10\ \Omega$, and V_o is unknown. If the current through R_2 is .24 amps, what are the currents through the other resistors and what is the voltage across the power supply?

For practice, try to do this problem on your own. Once you've tried, read on.

3.) Look at the circuit. What things do you know?

a.) You know you can combine R_3 and R_4 (they are in series) leaving you with one equivalent resistance $R_5 = 23\ \Omega$ (see Figure 13.34).

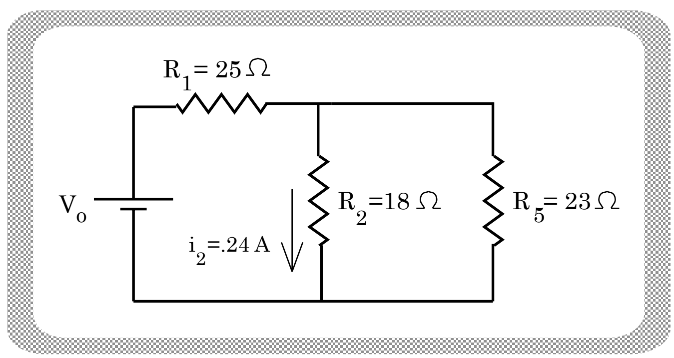


FIGURE 13.34

b.) You know you can use Ohm's Law to determine the voltage across R_2. That quantity is

$$\begin{aligned} V_2 &= i_2 R_2 \\ &= (.24\ \text{A})(18\ \Omega) \\ &= 4.32\ \text{volts}. \end{aligned}$$

c.) You know the voltage across R_2 will be the same as the voltage across R_5 (they are in parallel), or

$$V_2 = V_5$$
$$= 4.32 \text{ volts.}$$

d.) You know that Ohm's Law will allow you to determine the current through R_5. That is,

$$V_5 = i_5R_5$$
$$\Rightarrow \quad i_5 = \frac{V_5}{R_5}$$
$$= \frac{(4.32 \text{ volts})}{(23\ \Omega)}$$
$$= \frac{(4.32 \text{ volts})}{(23\ \Omega)}$$
$$= .188 \text{ amps.}$$

e.) You know the sum of the currents into the top node must equal the sum of the currents out of the top node (this is Kirchoff's First Law--see Figure 13.35), or

$$i_1 = i_2 + i_5$$
$$= (.24 \text{ A}) + (.188 \text{ A})$$
$$= .428 \text{ amps.}$$

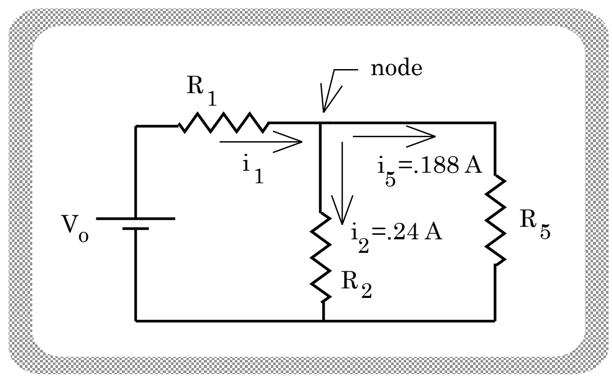


FIGURE 13.35

f.) From Ohm's Law, you know that the voltage across R_1 is

$$V_1 = i_1R_1$$
$$= (.428 \text{ A})(25\ \Omega)$$
$$= 10.7 \text{ volts.}$$

g) You know the sum of the voltage changes around the loop identified in Figure 13.36 must add to zero (this is Kirchoff's Second Law). That yields

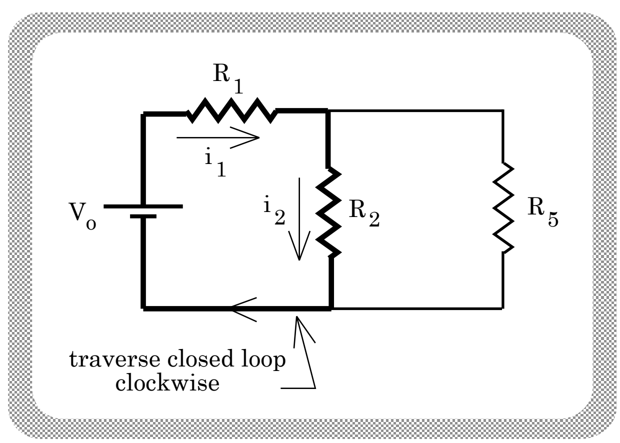


FIGURE 13.36

$$V_o - i_1R_1 - i_2R_2 = 0$$

$$\Rightarrow V_o = i_1R_1 + i_2R_2$$
$$= (.428 \text{ A})(25\ \Omega) + (.24 \text{ A})(18\ \Omega)$$
$$= 15 \text{ volts.}$$

ISN'T THIS FUN?

4.) Let's now look at the same set-up but with a different set of known values. For Figure 13.37, determine the voltmeter and ammeter readings when $R_1 = 25\ \Omega$, $R_2 = 18\ \Omega$, $R_3 = 13\ \Omega$, $R_4 = 10\ \Omega$, and $V_o = 15$ *volts*.

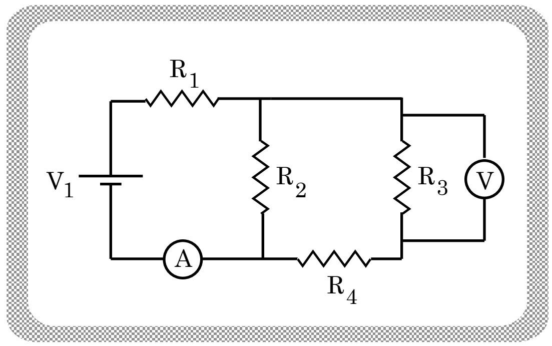


FIGURE 13.37

Note 1: This is a good example of a problem that can best be done using the formal approach associated with Kirchoff's Laws. That approach will be highlighted as we proceed through the problem.

Note 2: When you are asked to determine what an ammeter reads in a circuit diagram, you are obviously being asked to determine the *current* in the resistor's branch. What's interesting is that when you are asked to determine what a voltmeter across a resistor reads, you are also being asked to determine the *current* in the resistor's branch.

How so? By determining the current through a resistor, you can use Ohm's law (i.e., $V = iR$) to determine the voltage across the resistor (i.e., what does the voltmeter read?). Once you realize this, you no longer need the added complication of the meter stuck in the circuit. It is perfectly permissible to re-draw the circuit without the offending meter included.

a.) So, remove the meters for the sake of simplicity. Define current variables *for every branch* and identify all nodes in the circuit (see Figure 13.38). (Note that all currents are conventional.)

Note: The current i_3 in this problem has intentionally been defined in the wrong direction. Watch how the mathematics takes care of the error.

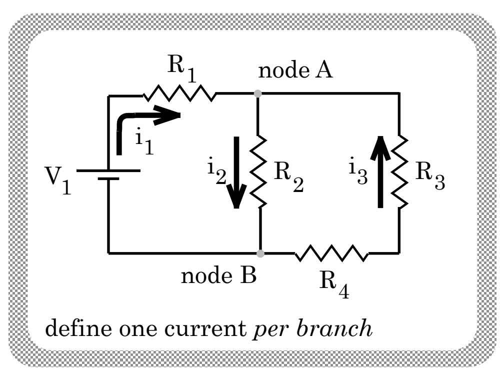


FIGURE 13.38

b.) *Kirchoff's First Law* written for Node A:

$$i_{in} = i_{out}$$
$$\Rightarrow \quad i_1 + i_3 = i_2.$$

Note: Satisfy yourself that the *node equation* written for *Node A* will give you the same information as the *node equation* written for *Node B* (this second equation is the above promised *duplicate expression*).

c.) Loop Equations: The loops are defined in Figure 13.39. We will need only two (there are three unknowns and we already have one equation courtesy of the node equation above).

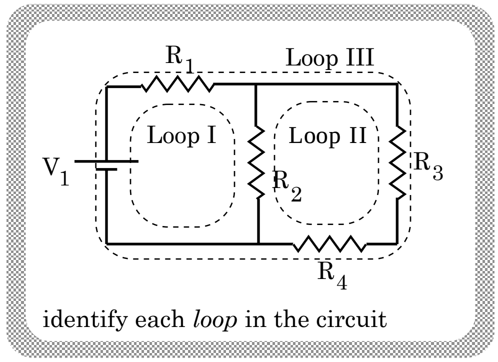


FIGURE 13.39

Note: Remember, the voltage DROPS (i.e., ΔV is *negative*) when one traverses through a resistor IN THE DIRECTION OF CONVENTIONAL CURRENT FLOW. The voltage INCREASES (i.e., ΔV is *positive*) when traversing through a resistor IN THE DIRECTION OPPOSITE CONVENTIONAL CURRENT FLOW.

Loop I: (traversing *clockwise* from Node B):

$$V_1 - i_1R_1 - i_2R_2 = 0$$
$$\Rightarrow \quad i_1 R_1 + i_2 R_2 = V_1$$
$$\Rightarrow \quad 25\, i_1 + 18\, i_2 = 15.$$

Loop II: (traversing *counterclockwise* from Node B):

$$- i_3R_4 - i_3R_3 - i_2R_2 = 0$$
$$\Rightarrow \quad i_2R_2 + i_3R_3 + i_3R_4 = 0$$
$$\Rightarrow \quad i_2R_2 + i_3(R_3 + R_4) = 0$$
$$\Rightarrow \quad 18\, i_2 + 23 i_3 = 0.$$

Note 1: Notice that if we had traversed Loop I *counterclockwise*, the Loop Equation would have read:

$$- V_1 + i_1R_1 + i_2R_2 = 0$$
$$\Rightarrow \quad i_1R_1 + i_2R_2 = V_1.$$

This is the same equation as was acquired by traversing clockwise.

Bottom line: You can traverse in any direction you wish. I usually try to move through batteries from ground to high voltage terminal because that gives positive voltage values for power supplies, but it really does not matter which way you do it as long as you keep your signs consistent.

Note 2: There are three loops in this circuit--the two used above and the one that moves around the outside of the circuit (Loop III). Although we don't need it, the third Loop Equation would have been:

$$V_1 - i_1R_1 + i_3 R_3 + i_3R_4 = 0.$$

Note 3: Just below we will be solving three equations simultaneously--two loop equations and one node equation. The temptation might be to forget the node equation and try to solve the three *loop equations*. **PLEASE NOTE**: The equation from Loop I added to the equation from Loop II gives us the equation from Loop III. That is, even though we have executed Kirchoff's Second Law on three loops, we have only two *INDEPENDENT* equations.

Bottom line: There will always be AT LEAST *one more node* and *one more loop* than there are *independent node equations* and/or *independent loop equations*. We will always have to use at least some node and some loop equations in solving circuit problems via simultaneous equations.

d.) Solve simultaneously:

$$i_1 + i_3 = i_2 \qquad \text{(Equation 1)}$$
$$25i_1 + 18i_2 = 15 \qquad \text{(Equation 2)}$$
$$18i_2 + 23i_3 = 0 \qquad \text{(Equation 3).}$$

Manipulating Equation 1 yields:

$$i_1 = i_2 - i_3 \qquad \text{(Equation 4).}$$

Substituting Equation 4 into Equation 2 yields:

$$25i_1 + 18i_2 = 15$$
$$25(i_2 - i_3) + 18i_2 = 15 \qquad \text{(Equation 5).}$$

Manipulating Equation 5:

$$43i_2 - 25i_3 = 15$$
$$\Rightarrow \quad i_2 = (25i_3 + 15)/43$$
$$= .58i_3 + .35 \qquad \text{(Equation 6).}$$

Substituting Equation 6 into Equation 3 yields:

$$18i_2 + 23i_3 = 0$$
$$18(.58i_3 + .35) + 23\, i_3 = 0 \qquad \text{(Equation 7).}$$

Manipulating Equation 7:

$$10.44i_3 + 6.3 + 23i_3 = 0$$
$$\Rightarrow \quad 33.4i_3 = -6.3$$
$$\Rightarrow \quad i_3 = -.189 \text{ amps.}$$

Note 1: The negative sign simply points out that the direction of the current i_3 was originally defined in the *wrong direction*. There is no need to change anything now--THE NEGATIVE SIGN SPEAKS FOR ITSELF.

Substituting back into Equation 6 yields:

$$i_2 = .58i_3 + .35$$
$$\Rightarrow \quad i_2 = .58\,(-.189) + .35$$
$$= .24 \text{ amps.}$$

Substituting back into Equation 1:

$$i_1 = i_2 - i_3$$
$$= .24 - (-.189)$$
$$= .43 \text{ amps.}$$

Note 2: These solutions are exactly the same as we determined using the less methodical "seat of your pants" technique outlined in the previous section.

e.) The solutions to the problem: a.) The *ammeter* will read the current i_1 which equals .43 amps, and b.) The *voltmeter* will read the voltage across R_3 which is i_3R_3 *= (.189 amps)(13 ohms) = 2.46 volts*.

Minor Technical Note: Voltmeters read magnitudes. If our voltmeter had been hooked up on the assumption that the current in R_3*'s* branch ran counterclockwise (that is what we assumed in the problem), the voltmeter would have been hooked up backwards and the meter's pointer would have been forced to move to the left off-scale instead of to the right on-scale.

Bottom line: In theory, the mathematics will take care of any mis-assumptions you may make about current directions. In lab, things are not so forgiving. That is why all meters used in lab must initially be set on their highest, least sensitive scale, and power to the circuit must be increased *slowly*.

N.) Matrix Approach to Analyzing Simultaneous Equations

1.) This section is not something you will, in all probability, use on a test. It does present an exotic way for you calculator wizards to solve simultaneous equations like the three found in *Equation 1*, *Equation 2*, and *Equation 3* above.

2.) Let's work with just three unknowns. Assume you have the equations:

$$(a)\, i_1 + (b)\, i_2 + (c)\, i_3 = V_1,$$
$$(d)\, i_1 + (e)\, i_2 + (f)\, i_3 = V_2,$$
$$(g)\, i_1 + (h)\, i_2 + (j)\, i_3 = V_3,$$

where i_1, i_2, and i_3 are unknown currents; the *a* through *j* terms are coefficients (positive or negative); and the *V* terms are voltages.

a.) Notice that we can display the coefficients as shown below.

a	b	c	V_1
d	e	f	V_2
g	h	j	V_3

b.) There are a number of things to note about this situation:

i.) Row #1 holds the *coefficients* of the first equation $(a)\, i_1 + (b)\, i_2 + (c)\, i_3 = V_1$, while;

ii.) Column #1 holds the i_1 coefficients for each of the equations. Likewise, column #2 holds the i_2 coefficients, column #3 holds the i_3 coefficients, and column #4 (this is actually a 1x3 matrix unto itself) holds the voltages.

c.) It may not be obvious at first glance, but this presentation of the *variable coefficients* is a natural matrix. Put in classical matrix notation, we end up with:

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & j \end{vmatrix} = \begin{vmatrix} V_1 \\ V_2 \\ V_3 \end{vmatrix}.$$

3.) If you have a calculator comparable to the TI-82 (or better), there is a tricky way you can solve for *all three* unknown currents at once (or, for that matter, as many unknown currents as you'd like). I'll prove the assertion below, but simply stated, it maintains the following:

a.) Enter the matrix (call it *A*) into your calculator.

b.) Have the calculator determine the *inverse matrix* A^{-1}.

c.) Multiply the inverse matrix A^{-1} by the constants matrix (this will be the single-column matrix in which the VOLTAGE terms have been placed).

d.) Your calculator will give you a single-column matrix. The value of the first entry in that matrix will numerically equal i_1, the second entry will numerically equal i_2, etc.

4.) Proof: Technically, our 3x3 matrix should be written as

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & j \end{vmatrix} \text{x} \begin{vmatrix} i_1 \\ i_2 \\ i_3 \end{vmatrix} = \begin{vmatrix} V_1 \\ V_2 \\ V_3 \end{vmatrix}$$, where we can shorthand the 3x3 part by calling it matrix *A* (that is, $A = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & j \end{vmatrix}$ so that $A \text{ x} \begin{vmatrix} i_1 \\ i_2 \\ i_3 \end{vmatrix} = \begin{vmatrix} V_1 \\ V_2 \\ V_3 \end{vmatrix}$). If we multiply both sides of our matrix equation by A^{-1}, we get:

$(A^{-1}\text{x}A)\text{x}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = A^{-1}\text{x}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$. Noting that $(A^{-1}\text{x}A) = 1$ (or, at least, a unit matrix), we get the expression $\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = A^{-1}\text{x}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$. Going back to our original assertion, if you multiply the inverse matrix A^{-1} by the single-column *constants* matrix (i.e., the matrix in which the voltages are found), you will end up with a matrix whose first term is numerically equal to the current associated with the first column of *A*, etc.

O.) An Alternate Matrix Approach to Analyzing Simultaneous Equations

1.) As stated in the previous section, a set of *n* algebraic expressions can be presented as an *n by n* matrix. As such, the *3x3* matrix shown below represents a system in which there are three independent equations.

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix}$$

Note 1: The matrix on the left is called the *determinate matrix D.*

Note 2: The evaluation of any matrix yields a number. The technique for doing such an evaluation has been or will be discussed in class. If the following explanation doesn't make perfect sense at first reading, don't panic. It will make sense sooner or later.

2.) To do a *pen to paper* evaluation of a 3x3 matrix, one must be able to evaluate a 2x2 matrix first. That is where we will begin.

a.) The evaluation of a 2x2 matrix follows:

$$\begin{bmatrix} o & p \\ q & s \end{bmatrix} = (os - pq),$$

where the elements *o*, *p*, *q*, and *s* can be either positive or negative.

That is, it is executed by subtracting the *product of the* *__upper right__* and *__bottom left__* entries from the *product of the* *__upper left__* and *__bottom right__* entries.

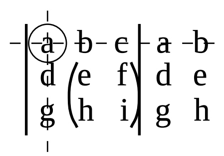
b.) Example of a 2x2 matrix evaluation: Evaluating the matrix presented below, we get:

$$\begin{bmatrix} -4 & 7 \\ -12 & 3 \end{bmatrix} = ((-4)(3) - (7)(-12)) = 72$$

4.) Consider now the 3x3 matrix shown below (it will become obvious shortly why the first and second columns are repeated to the right of the matrix). The evaluation of that matrix requires the sum of the evaluations of three 2x2 matrices. The technique follows:

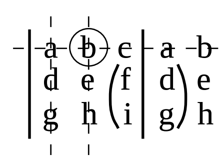
$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{matrix} a & b \\ d & e \\ g & h \end{matrix}$$

a.) Begin by drawing a line across the *top row* and down the *first column* of the 3x3 matrix. This will leave the *top-left* entry with a double-line through it:



b.) From the double-crossed entry (the *a* term in this case), there is a 2x2 matrix that starts in *the row just below* and the *column just to the right* of that entry (i.e., in the second row, second column). The first part of our 3x3 evaluation is the product of *a* times the evaluation of that 2x2 matrix, or:

$$a(ei - fh).$$



c.) Next, leaving the top row penciled out, draw a line down through the *second* column. This will leave the *middle-top* entry with a double-line through it (the *b* term in this case). From that double-crossed entry, there is a 2x2 matrix that starts in *the row just below* and the *column just to the right* of that entry (i.e., in the second row, *third* column).

The second part of our 3x3 evaluation is the product of *b* times the evaluation of that 2x2 matrix, or:

$$b(fg - di).$$

Note: It should now be obvious why the first column was repeated to the right of the matrix.

d.) Following a similar pattern, the third member of our 3x3 matrix evaluation is evaluated as:

$$c(dh - eg).$$

e.) The final evaluation is the sum of the three parts determined in *Sections b, c,* and *d* above. Mathematically, that is

$$a(ei - fh) + b(fg - di) + c(dh - eg).$$

5.) Let's assume we want to determine the *z* variable that satisfies our three equations but we do not care about the solutions for the variables *x* and *y*. There is a matrix technique that allows us to selectively solve for *z* while virtually ignoring *x* and *y*.

The technique maintains that the value of the *z* variable is equal to the *ratio of the evaluation of two matrices*--the *determinate matrix D* and a modified version of *the determinate*. Specifically:

$$z = D_{mod,z} / D.$$

a.) We have already defined the *determinate matrix D*. The *modified determinate matrix* is the *determinate matrix* <u>*with one change*</u>. The column associated with the variable for which we are solving (in this case, the *z* column) is replaced by the *constants column* from the original configuration (i.e., the 1x3 matrix that holds the constants in our equations). The modified matrix is shown below:

$$D_{mod,z} = \begin{bmatrix} a & b & m_1 \\ d & e & m_2 \\ g & h & m_3 \end{bmatrix}$$

b.) Mathematically, our *z* variable is equal to:

$$z = D_{mod,z} / D .$$

6.) An example with numbers. Assume:

$$
\begin{aligned}
x + 3y - z &= -4, \\
3x - 4y + 2z &= 12, \\
2x + 5y + 0z &= 9.
\end{aligned}
$$

Determine *z*.

a.) The *original matrix configuration* for this set of equations is:

$$
\begin{bmatrix} 1 & 3 & -1 \\ 3 & -4 & 2 \\ 2 & 5 & 0 \end{bmatrix} = \begin{bmatrix} -4 \\ 12 \\ 9 \end{bmatrix}
$$

b.) According to our technique:

$$
z = \frac{D_{mod,z}}{D} = \frac{\begin{bmatrix} 1 & 3 & -4 \\ 3 & -4 & 12 \\ 2 & 5 & 9 \end{bmatrix}}{\begin{bmatrix} 1 & 3 & -1 \\ 3 & -4 & 2 \\ 2 & 5 & 0 \end{bmatrix}}
$$

c.) The evaluation of these two matrices is:

$$
z = \frac{(1)[(-4)(9) - (12)(5)] + (3)[(12)(2) - (3)(9)] + (-4)[(3)(5) - (-4)(2)]}{(1)[(-4)(0) - (2)(5)] + (3)[(2)(2) - (3)(0)] + (-1)[(3)(5) - (-4)(2)]}
$$

$$
= \frac{(1)[-96]+(3)[-3]+(-4)[23]}{(1)[-10]+(3)[4]+(-1)[23]}
$$

$= (-197)/(-21)$

$= 9.38.$

7.) With this in mind, reconsider the two-battery circuit presented in the example in *Section H-2* of this chapter. The circuit is shown in Figure 16.30; the equations generated through Kirchoff's Laws were:

$$i_1 - i_2 - i_3 = 0 \qquad \text{(Equation 1)}$$
$$25\, i_1 + 18\, i_2 + 0 i_3 = 12 \qquad \text{(Equation 2)}$$
$$0 i_1 + 18\, i_2 - 20\, i_3 = 15 \qquad \text{(Equation 3).}$$

Note 1: The equations have been put in order in the sense that all the i_1 coefficients are in the first column, all the i_2 coefficients in the second column, etc.

Note 2: Even if there were, say, no i_2 coefficient in a particular equation, we would still need to acknowledge that spot in the matrix by placing a *zero* in the appropriate spot.

a.) Presenting and solving:

$$\begin{bmatrix} 1 & -1 & -1 \\ 25 & 18 & 0 \\ 0 & 18 & -20 \end{bmatrix} = \begin{bmatrix} 0 \\ 12 \\ 15 \end{bmatrix}$$

so that

$$i_1 = \frac{D_{mod,1}}{D} = \frac{\begin{bmatrix} 0 & -1 & -1 \\ 12 & 18 & 0 \\ 15 & 18 & -20 \end{bmatrix}}{\begin{bmatrix} 1 & -1 & -1 \\ 25 & 18 & 0 \\ 0 & 18 & -20 \end{bmatrix}}$$

b.) The evaluation of these two matrices is:

$$i_1 = \frac{(0)[(18)(-20) - (0)(18)] + (-1)[(0)(15) - (12)(-20)] + (-1)[(12)(18) - (18)(15)]}{(1)[(18)(-20) - (0)(18)] + (-1)[(0)(0) - (25)(-20)] + (-1)[(25)(18) - (18)(0)]}$$

$$= \frac{0 + (-1)[240] + (-1)[-54]}{1[-360] + (-1)[500] + (-1)[450]}$$

$= (186)/(1310)$

$= .14$ amps.

Note that this is the same value we calculated using the much more complicated *substitution method*.

8.) Bottom line: Kirchoff's Laws, in conjunction with the matrix approach we have been examining, are a very powerful tool for analyzing circuits in which only resistors and power supplies reside.

P.) Exotica--Real, Live Light Bulbs:

1.) There are a couple of fine points about electrical circuits that are brought out nicely when we examine real-life working light bulbs. We will do this through two separate scenarios.

2.) Scenario I: The resistor in the circuit shown in Figure 13.40 is a 40 watt household light bulb (note that household light bulbs require the equivalent of 120 volts DC to operate properly--that is, their RMS value is 120 volts). We will assume that the power supply in the circuit is a variable DC source.

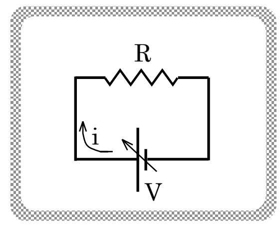


FIGURE 13.40

a.) To begin with, assuming we use *ideal meters* (i.e., an ammeter that has no resistance and a voltmeter whose resistance is infinite), there is absolutely no difference in the circuit shown in Figure 13.40 and the circuit shown in Figure 13.41.

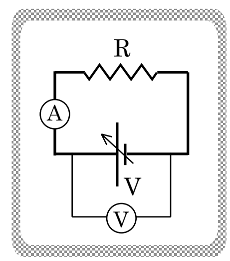


FIGURE 13.41

Note: This is a minor side point, but it never hurts to reinforce the fact that meters are theoretically assumed to do NOTHING in a circuit except sense whatever they are designed to measure. That's why I suggest that students re-draw diagrams *without the meters included* when confronted with a circuit they are expected to evaluate.

b.) Having made that observation, set the voltage across the variable power supply (hence the voltage across the light bulb) to 40 volts. Once done, examine your ammeter. Let's say it reads .20 amps.

c.) What should happen when you double the voltage to 80 volts?

i.) According to Ohm's Law, if you double the voltage across a resistor, the current through the resistor should double. The light bulb is acting like a resistor (or so we assume), so doubling its voltage to 80 volts should generate a doubled current of .40 amps.

ii.) In fact, if you try this in real life, you will find that the current goes up to only around .27 amps.

d.) So what's going on? It turns out that the *resistance* of a light bulb is dependent upon the temperature of the light bulb's filament.

i.) At low temperatures, there is very little vibratory motion of the atomic lattice through which charge carriers must flow. As a consequence, the carriers can travel fairly far (relatively speaking) before running into something.

ii.) At a macroscopic level, this long *mean free path* translates into *little resistance to current flow.*

iii.) At high temperatures, the atomic lattice through which charge carriers must flow experiences a lot of vibrational motion. As a consequence, charge carriers can't go very far (relatively speaking) without running into something.

iv.) At a macroscopic level, this short *mean free path* translates into *high resistance to current flow.*

Side Note: Although it is complicated, this partially explains why near-death light bulbs blow when they are being turned on, versus blowing after they've been on for a while. As was deduced above, a light bulb that is *off* has a very low filament temperature and, hence, low resistance to charge flow. When turned on, 120 volts are placed across the initially low resistance filament and a high current surges through the filament. This high current is what snaps the wire, sort of. Actually, this is a simplification. There's more to the story.

In fact, there are three reasons why bulbs blow. First, the filament physically thins down with time as atoms slough off the hot wire when at operating temperature. This thinning down makes the filament more vulnerable to breakage. Second, manufacturers who are not willing to wait for the natural demise of a thinning filament have dallied with planned obsolescence by putting a little bit of water vapor inside the bulb to corrode the filament with time. This also makes the filament more vulnerable to breakage. And third, the filament flexes as it expands due to heating when turned on and contracts when cooling after being turned off. With time, this expansion-contraction-expansion cycle fatigues the wire in the same way that the metal of a paper clip is fatigued when bent back and forth. At some point, the sudden expansion caused by the big initial current surge through a cold, low resistance, already fatigued and frail filament is what snaps the wire and kills the bulb.

e.) Back at the ranch, standard household light bulbs operate at the DC equivalent of 120 volts. So at 40 volts, the filament temperature is relatively low. When you double the voltage, the current and filament temperature will go up. As a consequence of the increase in temperature, the bulb's net *resistance* will also go up.

f.) In short, increasing the voltage will increase the current, but because the resistance has also gone up, the current won't go up as much as we might otherwise have expected. For that reason, in our example, the current went up from .2 amps to .27 amps instead of from .2 amps to .4 amps.

g.) Bottom line: Because a light bulb's resistance is temperature dependent, and because the filament temperature changes at different voltages, the current and voltage associated with a lit bulb are not linearly related. This is an example of a non-Ohmic device. Most electrical devices, from capacitors to inductors to transistors, are non-Ohmic in nature.

Note: On AP tests, unless stated otherwise, a light bulb is assumed to be an Ohmic device with a constant resistance value regardless of the current.

3.) Scenario II: Consider the parallel circuit shown in Figure 13.42. Assume the DC power supply is variable, and assume the resistors are two identical 40 watt light bulbs.

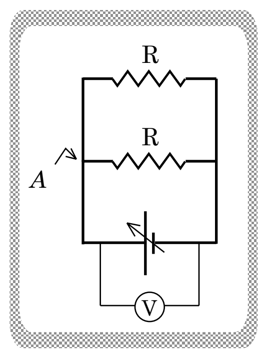


FIGURE 13.42

a.) To begin with, a standard, trick AP question is to present the circuit in Figure 13.42 and ask what would happen to the current through the top resistor if the middle resistor were removed from the circuit.

i.) Because the resistors are identical, most students observe that the current being drawn from the power supply (let's assume it is .6 amps) will split at the junction labeled *A* with half going through the middle resistor and the other half going through the top resistor. That's .3 amps through each resistor.

ii.) What a lot of students think is that when the middle resistor is removed, that full .6 amps will then flow through the top resistor. NOT SO!

iii.) What determines the amount of current that flows through a resistor is the size of the resistance and the *voltage across the resistor*. By taking the middle resistor out, you haven't changed the voltage *or* resistance of the top resistor. As a consequence, it will continue to draw the same amount of current it always drew--.3 amps in this case.

iv.) In other words, what changes in the circuit due to the removal of one of the parallel resistors is not the voltage or current through the remaining resistor. What changes is the amount of current being drawn from the battery. There is now one less path requiring current, so the battery current goes down.

b.) Having made that observation, consider the following: Set the voltage across the variable power supply in Figure 13.42 (hence the voltage across the two light bulbs) to 50 volts. Once accomplished, what will happen if you unscrew the middle light bulb?

i.) What you'd expect, according to our theory as presented, is that the upper light bulb would continue to burn with the same brightness as before. After all, nothing has changed in that branch of the circuit.

ii.) In fact, what happens if you try this is that the upper light bulb becomes MORE BRIGHT.

iii.) Furthermore, you will also find that the voltage across the power supply, as measured by the voltmeter in the circuit, will have *increased*. (When I tried this with my equipment at school, it went all the way up to 70 volts.)

d.) So what's going on? The key is in the fact that the voltage across the power supply seems to have gone up on its own. On the surface, this is very strange. Yet if you understand how power supplies really work, it makes sense. Follow along.

e.) Until now, all we have known about any power supply we've used has been what our voltmeter has told us. What voltmeters measure is called *the terminal voltage* of the source. That is, they measure the electrical potential difference between the terminals of the supply. This quantity is usually characterized as V, though for clarity, I will refer to it here as either V_{term} or $V_{terminal}$.

f.) What is often forgotten when treating a power supply as "ideal" is that there are *two* relevant, measurable parameters that make up a real power supply character.

g.) The first of these parameters is well known. It is the power supply's ability to provide energy to the circuit thereby motivating charge to flow. Power supplies do this internally by creating an electric field via an electrical potential difference across the supply's terminals.

i.) A true measure of the actual, energy-supplying, charge motivating aspect of the power supply is technically called the supply's *electromotive force*, or EMF. That quantity is usually characterized by an ε.

Note: This name happens to be a misnomer. The quality we are talking about here is not, as the name suggests, a force. It is a measure of the actual electrical potential difference *internally available* within the power supply. Its units are *volts* (note that these are the same units as the power supply's *terminal voltage*), not *newtons.*

h.) What has so-far been ignored is the second current-affecting quality associated with a power supply. That has to do with the power supply's internal resistance $r_{internal}$. (This will sometimes be characterized as r_i.) This internal resistance also creates a potential difference which is associated with the energy that is *lost* as current flows through the power source.

i.) This means that when we use a voltmeter to measure a power supply's *terminal voltage*, we are really measuring the energy-providing EMF (in volts) *MINUS* the energy-removing voltage drop due to the internal resistance (see Figure 13.43). This is formally expressed as:

$$V_{terminal} = \varepsilon - ir_{internal}.$$

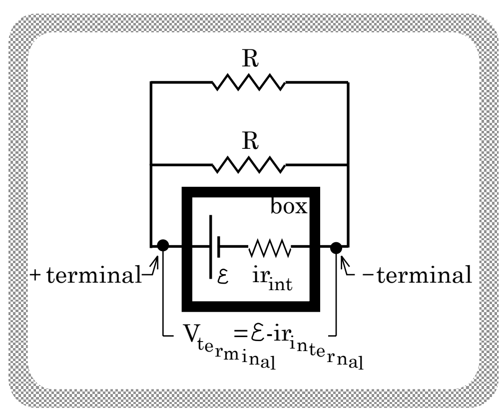


FIGURE 13.43

j.) With all of this in mind, what happened with our parallel circuit?

i.) Once the power supply's

EMF was initially set, it remained constant and didn't change.

ii.) There was a certain amount of net resistance wrapped up in the parallel combination of resistors coupled with the internal resistance of the power supply. The power supply generated the appropriate current for that net resistance and all was well.

iii.) One resistor was then removed.

iv.) With one less resistor drawing current, the current from the power supply went down.

v.) The EMF didn't change, but less current meant a smaller $ir_{internal}$ drop (if i goes down, ir goes down). That meant the terminal voltage $V_{terminal}$ went UP (look at the expression--if ε stays the same and $ir_{internal}$ goes down, then $V_{terminal}$ goes up).

vi.) This was why the terminal voltage in our scenario went from 50 volts to 70 volts.

vii.) Continuing, an increase in the terminal voltage increased the voltage across the remaining light bulb. That elicited a corresponding increase in current through that light bulb. That's why the bulb got brighter.

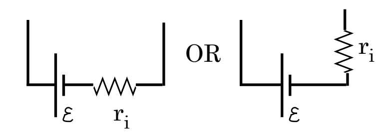
Note: Yes, when the current in the circuit goes up, there will be an increase in the ir_i drop which will, in turn, *decrease* the terminal voltage some . . . but not enough to counteract the terminal voltage *increase* due to the removal of the light bulb. It all works out in the end.

k.) So why is all of this being thrown at you?

i.) Most books talk about EMFs and internal resistance, but it usually seems like hair-splitting as they don't give concrete examples as to why you might want to care. Now you know how you can get messed up in at least one circumstance if you don't understand what is really going on inside a power source.

ii.) Standardized tests like the AP test will often give you a terminal voltage and nothing else. In that case, what they are telling you is that the power supply's internal resistance isn't going to play a part in the problem and you can forget it.

iii.) If, for whatever reason, the AP folks want you to consider the power supply's internal resistance, they will use the ε symbol for the battery's voltage and will include a resistor labeled r_i in series with the EMF value. Figure 13.44 depicts the two general ways this is done. It isn't a big deal. It simply means you have to treat that additional resistance--the internal resistance--as you would any other resistor in the circuit.



two ways internal resistance is characterized in a circuit

FIGURE 13.44

l.) So much for amusement.

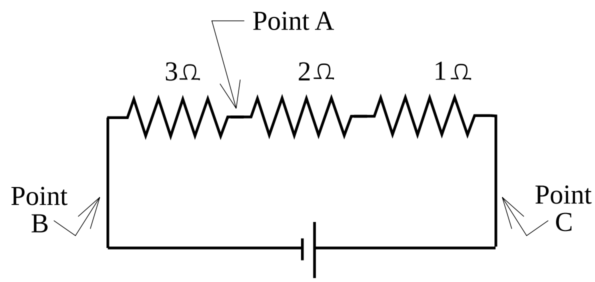
4.) Are there other elements that might be found in an electrical circuit? Yes, the capacitor, inductor, and semi-conductor devices. We will talk about all of them shortly.

QUESTIONS & PROBLEMS

13.1) What is the difference between voltage and current in a DC circuit?

13.2) A 12 ohm resistor has 2 amps of current passing through it. How much work does the resistor do on an electron moving through the resistor?

13.3) There are 3 amps of current being drawn from a power supply. The circuit is comprised of three resistors as shown.



- **a.)** In what direction will current as it is conventionally defined flow in this circuit?
- **b.)** In what direction will electrons flow in this circuit?
- **c.)** Some might suggest that there is a discrepancy between the answers to *Part a* and *b*. What is the problem, and why is this not really a discrepancy?
- **d.)** What is the voltage across the power supply?
- **e.)** What is the current at *Point A*?
- **f.)** What is the voltage drop across the 2 ohm resistor?
- **g.)** What is the net voltage across the 1 ohm and 2 ohm resistors (combined)?
- **h.)** How would the readings differ if you put an ammeter at *Point B* versus an ammeter at *Point C*?
- **i.)** If this were, in fact, an actual circuit you had built, how might you alter it if you wanted the current drawn from the power supply doubled? (You can assume it is an ideal power supply.)

13.4) Your friend has built a circuit. She says that when she *removes* one resistor in the circuit, the current drawn from the ideal power supply goes up.

- **a.)** What evidently happened to the effective resistance of the circuit when the resistor was removed?
- **b.)** Was the circuit a series or parallel combination?

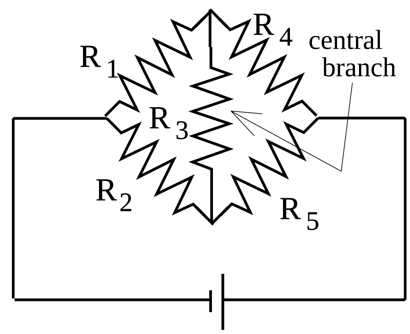
13.5) Your friend has built a circuit. He says that when he *adds* one resistor in the circuit, the current drawn from the ideal power supply goes up.

- **a.)** What evidently happened to the effective resistance of the circuit when the resistor was added?

b.) Was the circuit a series or parallel combination?

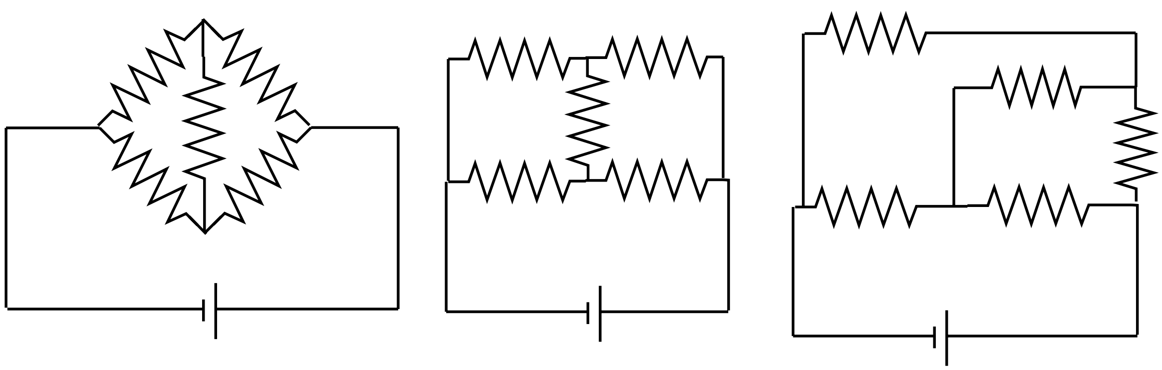
13.6) What is common in series connections?

13.7) What is common in parallel connections?



13.8) You find that the current through the central branch of the circuit to the right is *zero*. What *must* be true of the circuit for this to be the case?

13.9) If you had to determine the current being drawn from the power supply in at least one of the circuits shown below, which circuit would you pick? Explain.



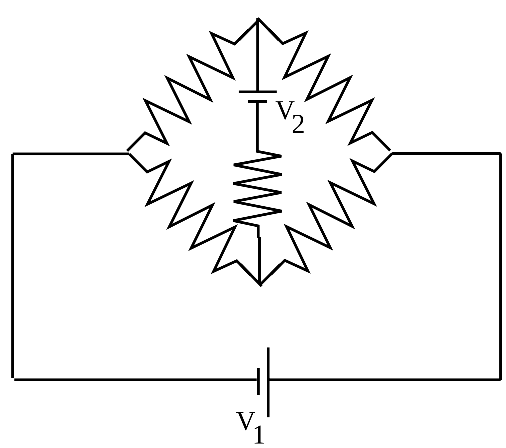
13.10) Charge carriers don't move very fast through electrical circuits. For instance, in a car's electrical system, charge moves at an incredibly slow 100 seconds per centimeter (that's a velocity of .01 cm/sec). So why do car lights illuminate immediately when you turn them on?

13.11) For the circuit shown to the right:

a.) How many nodes exist in the circuit?

b.) How many branches exist in the circuit?

c.) How many loops exist in the circuit?

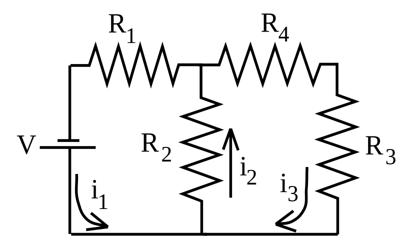


d.) How many equations would a rookie need to determine the current being drawn from the power supply V_1?

e.) What clever thing could you do that would halve the number of equations identified in *Part d*?

f.) There are two constraints placed on the equations you would need to solve for the current drawn from the power supply V_1. What are these constraints?

13.12) Consider the circuit shown to the right.



a.) How many nodes are there in this circuit?

b.) How many independent node equations can you write for this circuit?

c.) Why would you expect the number in *Part a* and the number in *Part b* to be different?

d.) There are three loops available in the circuit. If you wrote out loop equations for all three loops and tried to solve them simultaneously for the current i_2, you would get *zero* as a result. This makes no sense. What's wrong? That is, why are you calculating *zero current* in a section you *know* has current flowing through it?

e.) Are R_1 and R_2 in series? Justify.

f.) Are there *any* series combinations in the circuit?

g.) Are there *any* parallel combinations in the circuit?

h.) Is there anything wrong with the circuit as it is set up? That is, have I forgotten and/or mislabeled anything? Explain.

i.) Assume all the resistors have the same resistance, say, 10 ohms, except R_4, which is twice as large. Assume you don't know *V*, but you do know the voltage across R_2 is 60 volts. What is the *easiest* way to determine the voltage across R_4? In fact, for the humor of it, do that calculation.

13.13) The system of units generally in use in the U.S. is called *the English system of units*. What that means is that you grew up using measures like pounds, feet and inches as your standards. There are two units of measure in the world of electrical systems that you have grown up with that aren't a part of the English system of units. What are they?

13.14) Without changing anything else, you double the current through a resistor. How will that affect the power being dissipated by the resistor?

13.15) You have a resistor attached to an ideal power supply. You halve the resistance of the resistor. How will that affect the power being dissipated by the resistor?

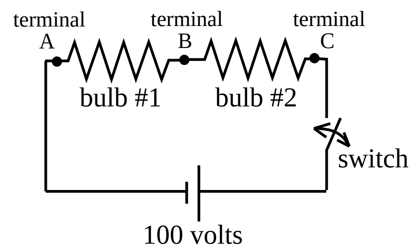
13.16) You have a resistor attached to a power supply. You halve the voltage of the ideal power supply.

a.) How will that affect the power being provided by the power supply?

b.) How will that affect the power being dissipated by the resistor?

13.17) You have a 10 watt light bulb and a 20 watt light bulb hooked in series in a circuit. Which bulb would you expect to have the greater resistance?

13.18) Two light bulbs are hooked in series to an ideal power supply (assume the resistance of a light bulb is *not* temperature dependent).



a.) *Bulb #1* is taken out of the circuit by putting a wire across terminals A and B. The switch is closed, and it is observed that *bulb #2* dissipates 40 watts of power.

i.) How much current is being drawn from the power supply in this situation?

ii.) What is the resistance of *bulb #2*?

b.) Removing the wire across terminals *A* and *B* and placing it across terminals *B* and *C*. The switch is closed and it is observed that *bulb #1* dissipates 10 watts of power.

i.) How much current is being drawn from the power supply in this situation?

ii.) What is the resistance of *bulb #1*?

c.) Both bulbs are placed in series across the power supply and the switch is thrown.

i.) What is the current in this circuit?

ii.) What will happen in the circuit? That is, will both bulbs light up? If not, which one won't . . . and why won't it?

d.) Using the terminals available in the sketch, can you add lines (i.e., wires) to make the original bulb configuration into a parallel combination without disconnecting any of the wires already there? If you can, draw the circuit. If you can't, explain what's stopping you.

e.) Let's say you did whatever was appropriate to make the bulb configuration into a parallel combination.

i.) How will the current in the circuit change from what it was as a series combination? Think about this conceptually before trying to do it mathematically.

ii.) Is more power dissipated in the series or the parallel combination? Again, think about this conceptually.

iii.) What is the ratio of power between the two kinds of circuits?

13.19) Consider Figure I:

a.) If no current is to flow through R_5, what must be true?

b.) If $V_A = 3.36$ *volts* and $V_B = 5.25$ *volts* (*Points A* and *B* are defined in the sketch), in which direction will current flow through R_5?

c.) Given the situation outlined in *Part b* and assuming $R_5 = 3\ \Omega$, what is the current through R_5?

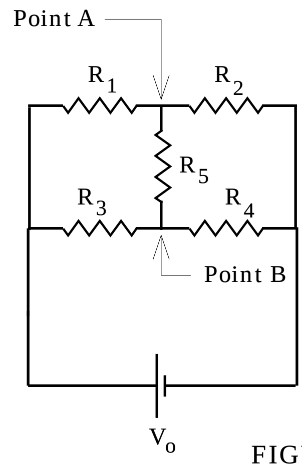


FIGURE I

13.20) In the circuit in Figure II, the current through the 12 Ω resistor is .5 amps.

a.) What is the current through the 8 Ω resistor?

b.) What is the power supply's voltage?

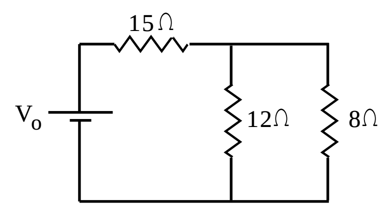


FIGURE II

13.21) In Figure III, R_2 is decreased. Assuming an ideal power supply, what happens to:

a.) R_2's voltage;

b.) R_2's current;

c.) R_1's voltage;

d.) the power dissipated by R_2?

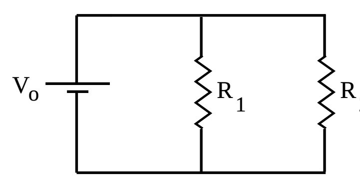


FIGURE III

13.22) In Figure IV, all the resistors are the same and all the ideal power sources are the same. If the current in the series circuit is .4 amps, what is the current drawn from V_o in the parallel circuit?

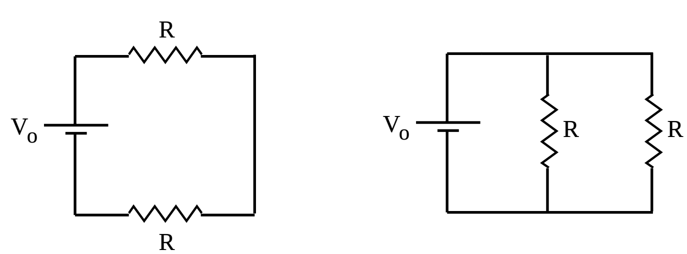


FIGURE IV

13.23) Resistors $R_1 = 10\ \Omega$, $R_2 = 12\ \Omega$, and $R_3 = 16\ \Omega$ are connected in parallel. If the current through the 12 Ω resistor is 2 amps, determine the currents through the other two resistors.

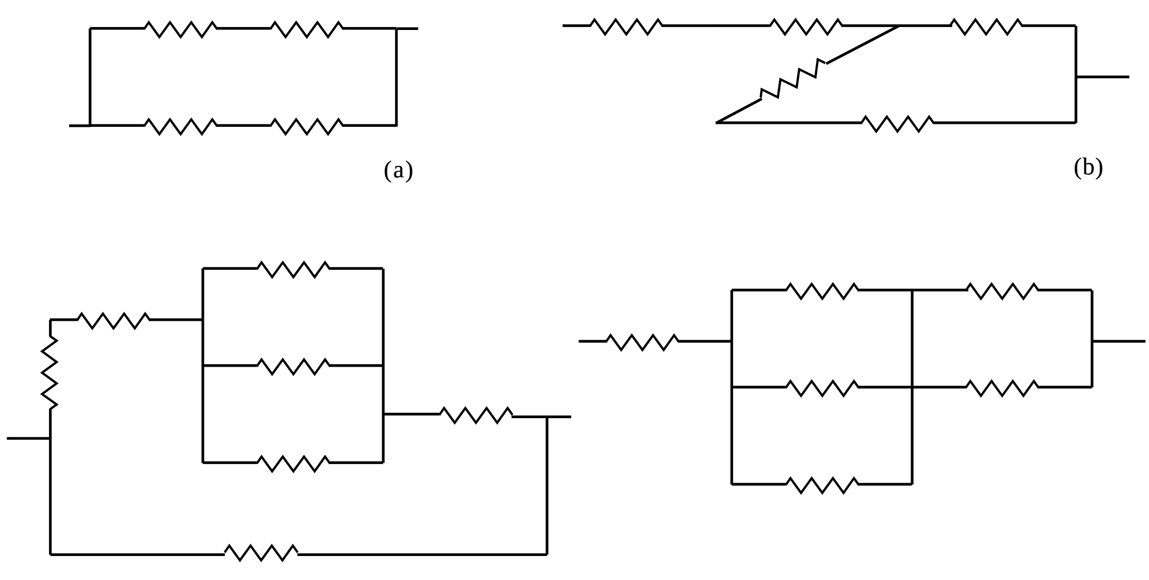
13.24) A battery charger delivers 6 amps of current to a 45 Ω resistor for 30 minutes.

- **a.)** How much charge passes through the resistor?
- **b.)** How much work does the charger do?
- **c.)** How much power does the charger deliver?

13.25) A power supply provides 125 watts to an 18 Ω resistor. Determine:

- **a.)** The current through the resistor; and
- **b.)** The voltage across the resistor.

13.26) Assuming all resistors available are 5 Ω, determine the equivalent resistance of the circuits found in Figure V.



(a) (b) (c) (d)

FIGURE V

13.27) Using as many 12 Ω resistors as you need, produce a resistor circuit whose equivalent resistance is:

a.) 18 Ω; and

b.) 30 Ω.

13.28) The ideal power dissipated by the circuit in Figure VI is 800 watts. What is *R*?

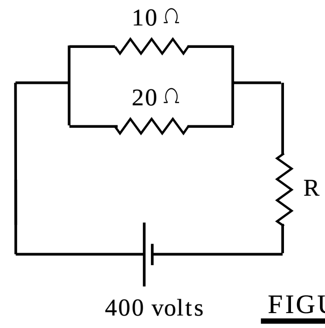


FIGURE VI

13.29) Examine Figure VII:

a.) How many nodes are there in the circuit?

b.) How many loops?

c.) Write out any three *node equations* using the information provided in the circuit.

d.) Write out any three *loop equations*.

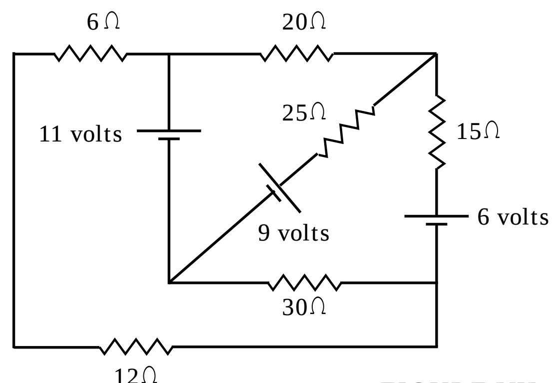


FIGURE VII

13.30) Use Kirchoff's Laws to determine the meter readings in the circuits shown in Figure VIII.

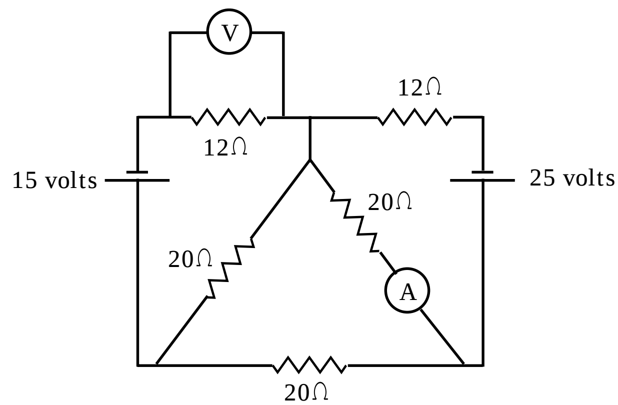


FIGURE VIII

Note 1: There are *five primary branches* in the circuit (primary branches do not include *voltmeter branches*). That means you are either going to have to analyze a 5x5 matrix or be clever in the way you define your currents. My suggestion: be clever. Begin by defining a current or two, then use your node equations to define all the other currents in terms of the first few. In doing so, you should be able to whittle the number of variables down considerably.

13.31) Charge carriers in a DC circuit move in one direction only. What do charge carriers do in an AC circuit?

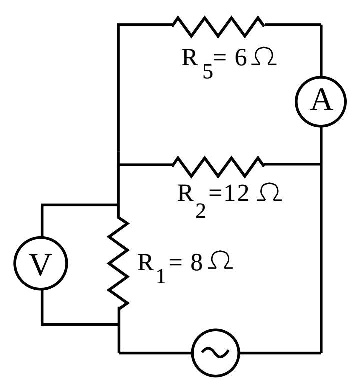
13.32) The idea of *current through* and *voltage across* a resistor in a DC circuit is fairly straightforward. Current measures the amount of charge that passes through the resistor *per unit time*, and voltage measures the unchanging voltage difference between the two sides of the resistor. The idea of a resistor's current and voltage in an AC circuit is a little more complex, given the fact that the charge carriers in AC circuits don't really go anywhere. So how do we deal with the idea of current and voltage in an AC circuit? That is, when someone says that your home wall socket is providing 110 volts AC, and that a light bulb plugged into that socket draws .2 amps of current, what are those numbers really telling you?

13.33) An AC voltage source is found to produce a 12 volt peak to peak signal at 2500 hertz.

- **a.)** Characterize this voltage as a *sine function*.
- **b.)** Graph the *voltage versus time* function.
- **c.)** Determine the *RMS voltage* of the source and put that value on your graph from *part b*.
- **d.)** It is found that an ammeter in the circuit reads 1.2 amps. What is the *maximum current* drawn from the source?

13.34) The AC source shown in the circuit provides a voltage equal to 25 sin (300t).

- **a.)** What is the frequency of the source?
- **b.)** What will the voltmeter read in the circuit?
- **c.)** What will the ammeter read in the circuit?

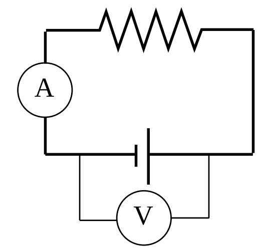


13.35) The circuit shown in *problem 13.34* is plugged into a WALL SOCKET.

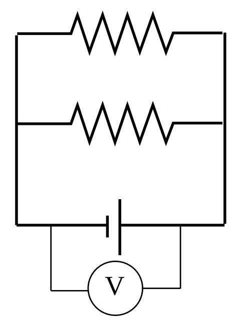
- **a.)** What is the frequency of the source?
- **b.)** Characterize this voltage as a *sine function*.
- **c.)** What will the voltmeter read in the circuit?
- **d.)** What will the ammeter read in the circuit?

13.36) The first voltmeter you used in the circuit shown in Problem 13.34 didn't register anything. It wasn't broken. What was likely the problem?

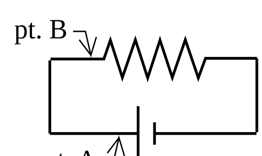
13.37) I used a transformer to step up the wall-socket voltage (AC) to 5000 volts so that I could charge up a 6000 volt capacitor. After making the voltage into DC (I used a rectifier to do this), I managed to charge the capacitor . . . to the point of blowing it up (well, it didn't blow, but it did die). What went wrong? What was I NOT taking into consideration when I bought the transformer?



13.38) The theory is simple. Attach a power supply to a resistor. Put a voltmeter across the power supply to measure the supply's voltage and put an ammeter into the circuit measure the current through the resistor. According to Ohm's Law, if you double the voltage, the current should double. So, our hapless teacher takes a light bulb and attaches it to a variable DC power supply (see sketch). She uses the voltmeter to set the power supply voltage (hence the voltage across the bulb) at 40 volts. At this setting the ammeter reads .45 amps. She doubles the voltage to 80 volts expecting to see the current double to .9 amps. What she sees instead is that the current has risen to only .65 amps. So there she sits before a classroom full of students looking like Jacqueline the idiot, with no explanation to be had. Is Ohm's Law working here? Why is the circuit acting the way it is?



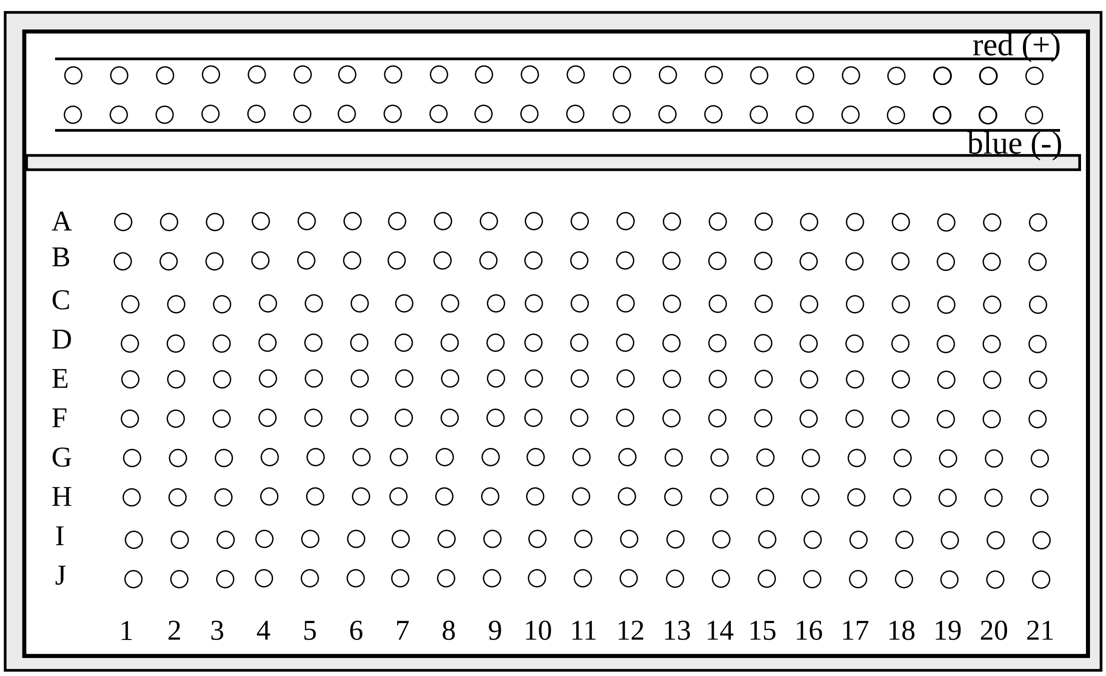
13.39) Once again, the theory is simple. Attach a power supply to two resistors connected in parallel. The voltage across each will be the same and will equal, to a very good approximation, the voltage across the power supply. In theory, nothing will happen to the current through either of the resistors if the other resistors is removed. Why? The voltage will not change across the remaining resistor, so the current should still be V_R/R. So, our hapless teacher (this time a male) takes two light bulbs, hooks them in parallel across a DC power supply, connects a voltmeter across the power supply and uses the meter to set the voltage at 80 volts. He then unscrews one of the light bulbs expecting to find that the brightness of the second bulb does not change. What he finds is that it gets brighter. So there he sits before a classroom full of students looking like Jack the idiot, with no explanation to be had. Is Ohm's Law working here? In any case, why is the circuit acting the way it is?



13.40) You hook up the simple circuit shown to the right. Assume you make the resistance of the resistor really big. You muse about the circuit's inner workings. You realize that an electron leaving the battery has a certain amount of energy. You acknowledge that the voltage at *A* and the voltage at *B* are essentially the same, so you accept that only a little bit of the electron's energy will be lost as it moves through the wire. In fact, almost all of its energy will be lost as the electron passes through the resistor. It all makes sense . . . until.

Being perverse, you remove the larger resistor and replace it with a resistor of smaller resistance. You muse. Your guinea pig electron still has the same amount of energy as it leaves the battery, and it still loses the same amount of energy as it passes through the wire, but now it has to get rid of almost all of its energy through a much smaller resistance. How does that work? How can the size of the resistor not matter when it comes to energy loss?

13.41) On the breadboard provided, show the wiring for two resistors and a power supply in series with three resistors in parallel (assume numbers are common).



13.42) On the breadboard provided, show the wiring for a power supply in series with three groups of two parallel resistors, all in parallel with a single resistor.